Unit-12

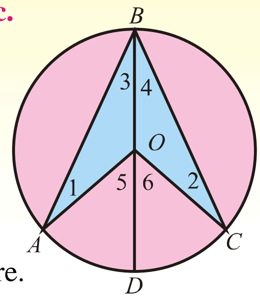
ANGLE IN A SEGMENT OF A CIRCLE

In this unit, students will learn

- ***The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.***
- ***Any two angles in the same segment of a circle are equal.***
- ***The angle***
  - ***in a semi-circle is a right angle,***
  - ***in a segment greater than a semi circle is less than a right angle,***
  - ***in a segment less than a semi-circle is greater than a right angle.***
- ***The opposite angles of any quadrilateral inscribed in a circle are supplementary.***

THEOREM 1

12.1(*i*) The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.



Given: $\overset{\frown}{AC}$ is an arc of a circle with centre O.

Whereas $\angle AOC$ is the central angle

and $\angle ABC$ is circum angle.

To prove: $m\angle AOC = 2m\angle ABC$

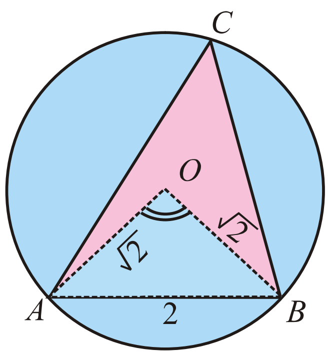
Construction: Join B with O and produce it to meet the circle at D.

Write angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ $\angle 5$ and $\angle 6$ as shown in the figure.

Proof:

Statements	Reasons
As $m\angle 1 = m\angle 3$ (i)	Angles opposite to equal sides in ΔOAB
and $m\angle 2 = m\angle 4$ (ii)	Angles opposite to equal sides in ΔOBC
Now $m\angle 5 = m\angle 1 + m\angle 3$ (iii)	External angle is the sum of internal opposite angles.
Similarly $m\angle 6 = m\angle 2 + m\angle 4$ (iv)	
Again $m\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3$ (v)	Using (i) and (iii)
and $m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4$ (vi)	Using (ii) and (iv)
Then from figure	
$\Rightarrow$ $m\angle 5 + m\angle 6 = 2m\angle 3 + 2m\angle 4$	Adding (v) and (vi)
$\Rightarrow$ $m\angle AOC = 2(m\angle 3 + m\angle 4) = 2m\angle ABC$	

Example: The radius of a circle is $\sqrt{2}$ cm. A chord 2 cm in length divides the circle into two segments. Prove that the angle of larger segment is 45°.



Given: In a circle with centre O and radius $m\,\overline{OA} = m\,\overline{OB} = \sqrt{2}$ cm,

The length of chord $\overline{AB} = 2$ cm divides the circle into two segments with ACB as larger one.

To prove: $m\angle ACB = 45^o$

Construction: Join O with A and O with B.

Proof:

Statements	Reasons
In $\Delta\ OAB$	$m\overline{OA} = m\overline{OB} = \sqrt{2}$ cm
$(OA)^2 + (OB)^2 = (\sqrt{2})^2 + (\sqrt{2})^2$	
$= 2 + 2 = 4$	
$= (2)^2 = (AB)^2$	Given $m\ \overline{AB} = 2$ cm
$\therefore$ ΔAOB is right angled triangle	
With $m\ \angle AOB = 90^o$	Which being a central angle standing on an arc AB
Then $m\angle ACB = \frac{1}{2} m\angle AOB$	By theorem 1
$= \frac{1}{2}(90^o) = 45^o$	Circum angle is half of the central angle.

THEOREM 2

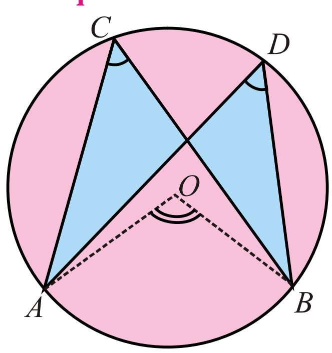
12.1(ii) Any two angles in the same segment of a circle are equal.

Given: $\angle\ ACB$ and $\angle\ ADB$ are the circum angles in the same segment of a circle with centre O.

To prove: $m\ \angle ACB = m\ \angle ADB$

Construction: Join O with A and O with B.

So that $\angle AOB$ is the central angle.



Proof:

Statements	Reasons
Standing on the same arc AB of a circle.	
$\angle AOB$ is the central angle whereas	Construction
$\angle ACB$ and $\angle ADB$ are circum angles	Given
$\therefore$ $m\angle AOB = 2m\angle ACB$ (i)	By theorem 1
and $m\angle AOB = 2m\angle ADB$ (ii)	By theorem 1
$\Rightarrow$ $2m\angle ACB = 2m\angle ADB$	Using (i) and (ii)
Hence, $m\angle ACB = m\angle ADB$	

THEOREM 3

12.1(*iii*) The angle

- **in a semi-circle is a right angle,**
- **in a segment greater than a semi circle is less than a right angle,**
- **in a segment less than a semi-circle is greater than a right angle.**

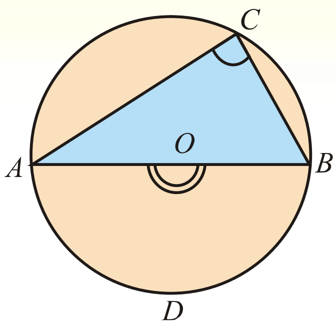


Fig. I

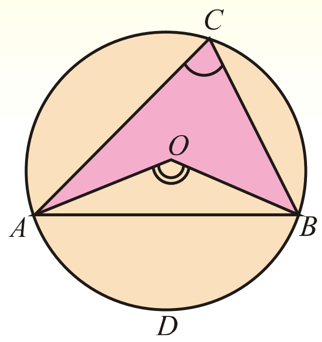


Fig. II

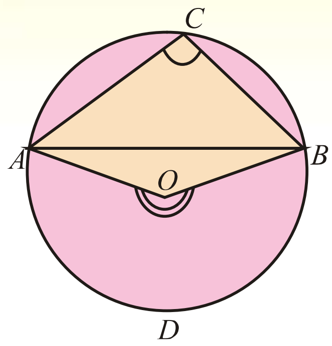


Fig. III

Given: $\overline{AB}$ is the chord corresponding to an arc ADB
Whereas $\angle AOB$ is a central angle and $\angle ACB$ is
a circum angle of a circle with centre O.

To prove: In fig (I) If sector ACB is a semi circle
then $m\angle ACB = 1\angle rt$

In fig (II) If sector ACB is greater than a semi circle
then $m\angle ACB < 1\angle rt$

In fig (III) If sector ACB is less than a semi circle
then $m\angle ACB > 1\angle rt$

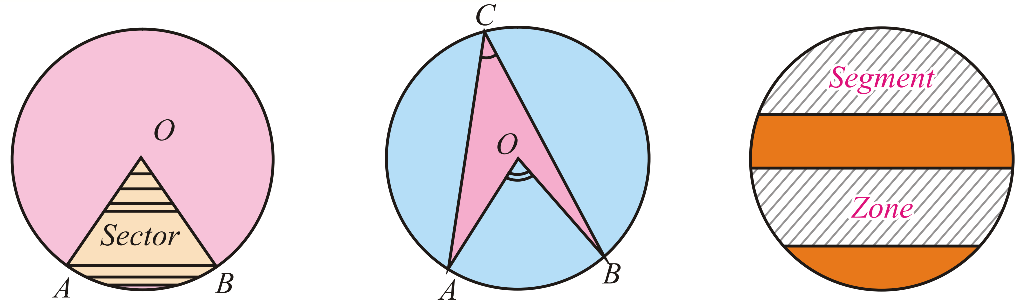
Proof:

Statements	Reasons
In each figure, $\overline{AB}$ is the chord of a circle with centre O.	Given
$\angle AOB$ is the central angle standing on an arc ADB. Whereas $\angle ACB$ is the circum angle	Given
Such that $m\angle AOB = 2m\angle ACB$ (i)	By theorem 1
Now in fig (I) $m\angle AOB = 180^o$	A straight angle
$\therefore$ $m\angle AOB = 2\angle rt$ (ii)	
$\Rightarrow$ $m\angle ACB = 1\angle rt$	Using (i) and (ii)
In fig (II) $m\angle AOB < 180^o$	
$\therefore$ $m\angle AOB < 2\angle rt$ (iii)	

$\Rightarrow$	$m\angle ACB < 1\angle rt$		Using (i) and (iii)
In fig (III)	$m\angle AOB > 180^\circ$		
$\therefore$	$m\angle AOB > 2\angle rt$	(iv)	
$\Rightarrow$	$2m\angle ACB > 2\angle rt$		Using (i) and (iv)
$\Rightarrow$	$m\angle ACB > 1\angle rt$		

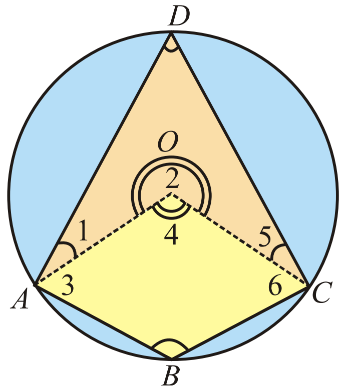
Corollary 1. The angles subtended by an arc at the circumference of a circle are equal.

Corollary 2. The angles in the same segment of a circle are congruent.



THEOREM 4

12.1(*iv*) The opposite angles of any quadrilateral inscribed in a circle are supplementary.



Given: $ABCD$ is a quadrilateral inscribed in a circle with centre O.

To prove: $\begin{cases} m\angle A + m\angle C = 2\ \angle rts \\ m\angle B + m\angle D = 2\ \angle rts \end{cases}$

Construction: Draw $\overline{OA}$ and $\overline{OC}$.

Write $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ as shown in the figure.

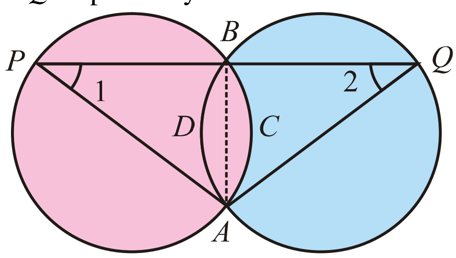
Proof:

Statements	Reasons
Standing on the same arc ADC, $\angle 2$ is a central angle Whereas $\angle B$ is the circum angle	Arc ADC of the circle with centre O.
$\therefore \quad m\angle B = \frac{1}{2}(m\angle 2) \quad$ (i)	By theorem 1
Standing on the same arc ABC, $\angle 4$ is a central angle whereas $\angle D$ is the circum angle	Arc ABC of the circle with centre O.
$\therefore \quad m\angle D = \frac{1}{2}(m\angle 4) \quad$ (ii)	By theorem 1
$\Rightarrow \quad m\angle B + m\angle D = \frac{1}{2}m\angle 2 + \frac{1}{2}m\angle 4$	Adding (i) and (ii)
$= \frac{1}{2}(m\angle 2 + m\angle 4) = \frac{1}{2}$(Total central angle)	
i.e., $\quad m\angle B + m\angle D = \frac{1}{2}(4\angle rt) = 2\angle rt$	
Similarly $m\angle A + m\angle C = 2\angle rt$	

Corollary 1. In equal circles or in the same circle if two minor arcs are equal then angles inscribed by their corresponding major arcs are also equal.

Corollary 2. In equal circles or in the same circle, two equal arcs subtend equal angles at the circumference and vice versa.

Example 1: Two equal circles intersect in A and B. Through B, a straight line is drawn to meet the circumferences at P and Q respectively. Prove that $m\overline{AP} = m\overline{AQ}$.



Given: Two equal circles cut each other at points A and B. A straight line PBQ drawn through B meets the circles at P and Q respectively.

To prove: $m\overline{AP} = m\overline{AQ}$

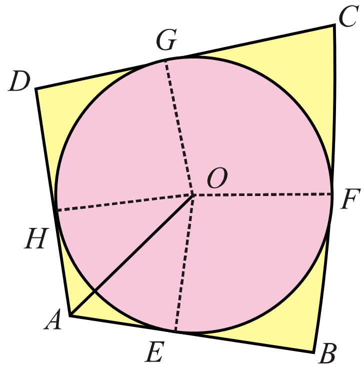
Construction: Join the points A and B. Also draw $\overline{AP}$ and $\overline{AQ}$.
Write $\angle 1$ and $\angle 2$ as shown in the figure.

Proof:

Statements	Reasons
	Arcs about the common chord AB.
$\because \quad m\overparen{ACB} = m\overparen{ADB}$	
$\therefore \quad m\angle 1 = m\angle 2$	Corresponding angles made by opposite arcs.
So $\quad m\overline{AQ} = m\overline{AP}$	Sides opposite to equal angles in $\Delta\, APQ$.
or $\quad m\overline{AP} = m\overline{AQ}$	

Example 2: $ABCD$ is a quadrilateral circumscribed about a circle.

Show that $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$



Given: $ABCD$ is a quadrilateral circumscribed about a circle with centre O.

So that each side becomes tangent to the circle.

To prove: $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$

Construction: Drawn $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{BC}$

$\overline{OG} \perp \overline{CD}$ and $\overline{OH} \perp \overline{DA}$

Proof:

Statements	Reasons
$\therefore \quad m\overline{AE} = m\overline{HA}$; $m\overline{EB} = m\overline{BF}$ ... (i)	Since tangents drawn from a point to the circle are equal in length
$m\overline{CG} = m\overline{FC}$ and $m\overline{GD} = m\overline{DH}$ ... (ii)	
$(m\overline{AE} + m\overline{EB}) + (m\overline{CG} + m\overline{GD}) = (m\overline{BF} + m\overline{FC}) + (m\overline{DH} + m\overline{HA})$	Adding (i) & (ii)
or $\quad m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$	

EXERCISE 12.1

1. Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.
2. Show that parallelogram inscribed in a circle will be a rectangle.
3. AOB and COD are two intersecting chords of a circle. Show that Δ^s AOD and BOC are equiangular.
4. $\overline{AD}$, and $\overline{BC}$ are two parallel chords of a circle. Prove that arc $AB \cong$ arc CD and arc $AC \cong$ arc BD.

MISCELLANEOUS EXERCISE 12

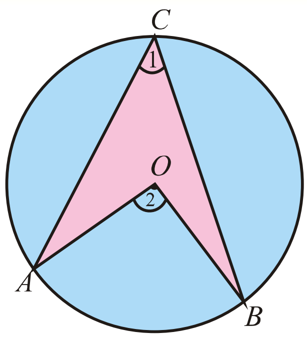
1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(*i*) A circle passes through the vertices of a right angled ΔABC with $m\overline{AC} = 3$cm and $m\overline{BC} = 4$ cm, $m\angle C = 90^{\circ}$. Radius of the circle is:

(*a*) 1.5 cm (*b*) 2.0 cm (*c*) 2.5 cm (*d*) 3.5 cm

(*ii*) In the adjacent circular figure, central and inscribed angles stand on the same arc *AB*. Then



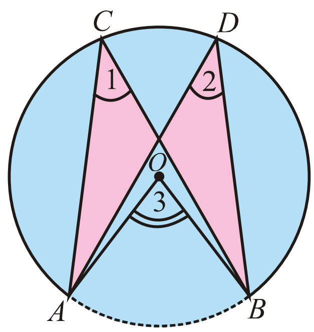
(*a*) $m\angle 1 = m\angle 2$ (*b*) $m\angle 1 = 2m\angle 2$

(*c*) $m\angle 2 = 3m\angle 1$ (*d*) $m\angle 2 = 2m\angle 1$

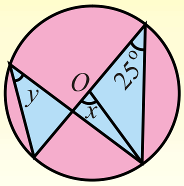
(*iii*) In the adjacent figure if $m\angle 3 = 75^{\circ}$, then find $m\angle 1$ and $m\angle 2$.

(*a*) $37\frac{1}{2}^{\circ}, 37\frac{1}{2}^{\circ}$ (*b*) $37\frac{1}{2}^{\circ}, 75^{\circ}$

(*c*) $75^{\circ}, 37\frac{1}{2}^{\circ}$ (*d*) $75^{\circ}, 75^{\circ}$

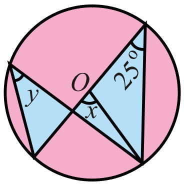


(*iv*) Given that O is the centre of the circle. The angle marked x will be:



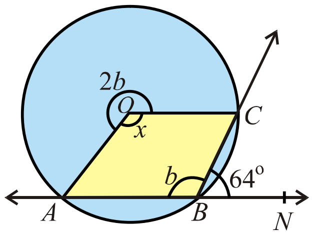
(*a*) $12\frac{1}{2}^{\circ}$ (*b*) 25° (*c*) 50° (*d*) 75°

(*v*) Given that O is the centre of the circle the angle marked y will be:



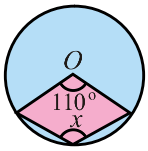
(*a*) $12\frac{1}{2}^{\circ}$ (*b*) 25° (*c*) 50° (*d*) 75°

(*vi*) In the figure, O is the centre of the circle and $\overleftrightarrow{ABN}$ is a straight line. The obtuse angle $AOC = x$ is:



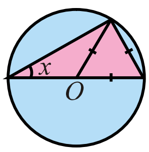
(*a*) 32° (*b*) 64° (*c*) 96° (*d*) 128°

(*vii*) In the figure, O is the centre of the circle, then the angle x is:



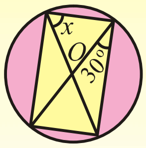
(*a*) 55° (*b*) 110° (*c*) 220° (*d*) 125°

(*viii*) In the figure, O is the centre of the circle then angle x is:



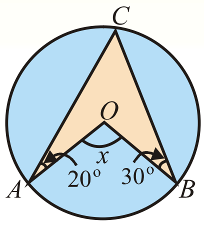
(*a*) 15° (*b*) 30° (*c*) 45° (*d*) 60°

(*ix*) In the figure, O is the centre of the circle then the angle x is:



(*a*) 15° (*b*) 30° (*c*) 45° (*d*) 60°

(*x*) In the figure, O is the centre of the circle then the angle x is:



(*a*) 50° (*b*) 75° (*c*) 100° (*d*) 125°

SUMMARY

- The angle subtended by an arc at the centre of a circle is called is **central angle**.
- A **central angle** is subtended by two radii with the vertex at the centre of the circle.
- The angle subtended by an arc of a circle at its circumference is called a **circumangle**.
- A **circumangle** is subtended between any two chords of a circle, having common point on its circumference.
- A quadrilateral is called **cyclic** when a circle can be drawn through its four vertices.
- The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
- Any two angles in the same segment of a circle are equal.
- The angle
  - in a semi-circle is a right angle,
  - in a segment greater than a semi-circle is less than a right angle,
  - in a segment less than a semi-circle is greater than a right angle.
- The opposite angles of any quadrilateral inscribed in a circle are supplementary.