Unit-13

PRACTICAL GEOMETRY-CIRCLES

In this unit, students will learn how to

- *locate the centre of a given circle.*
- *draw a circle passing through three given non-collinear points.*
- *complete the circle when a part of its circumference is given,*
  (i) by finding the centre,
  (ii) without finding the centre.
- *circumscribe a circle about a given triangle.*
- *inscribe a circle in a given triangle.*
- *escribe a circle in a given triangle.*
- *circumscribe an equilateral triangle about a given circle.*
- *inscribe an equilateral triangle in a given circle.*
- *circumscribe a square about a given circle.*
- *inscribe a square in a given circle.*
- *circumscribe a regular hexagon about a given circle.*
- *inscribe a regular hexagon in a given circle.*
- *draw a tangent to a given arc, without using the centre, through a given point p when p is the middle point of the arc, p is at the end of the arc and p is outside the arc.*
- *draw a tangent to a given circle from a point P when P is on the circumference and when p is outside the circle.*
- *draw two tangents to a circle meeting each other at a given angle.*
- *draw direct common tangent or external tangents to two equal circles and draw transverse common tangents or internal tangents to two equal circles.*
- *draw direct common tangents or external tangents to two unequal circles and draw transverse common tangents or internal tangents to two unequal circles.*
- *draw a tangent to two unequal touching circles and two unequal intersecting circles.*
- *draw a circle which touches*
  (i) both the arms of a given angle.
  (ii) two converging lines and passes through a given point between them.
  (iii) three converging lines.

INTRODUCTION:

The word geometry is derived from two Greek words namely Geo (earth) and Metron (measurement). Infact, geometry means measurement of the earth or land. Geometry is an important branch of mathematics, which deals with the shape, size and position of geometric figures. We will concentrate upon simple figures namely point, straight line, triangle, polygon and circle in this unit.

The Greek mathematicians (600-300 BC) contributed a lot. In particular "Euclid's Elements" have been taught as text book all over the world for centuries.

13.1 Construction of a Circle

A circle of any radius can be constructed by rotating a compass about a fixed point O.

13.1(i) To locate the centre of a given circle

Given: A circle

Steps of Construction:

1. Draw two chords $\overline{AB}$ and $\overline{CD}$.
2. Draw $\overleftrightarrow{EFG}$ as perpendicular bisector of chord $\overline{AB}$.
3. Draw $\overleftrightarrow{PQR}$ as perpendicular bisector of chord $\overline{CD}$.
4. Perpendicular bisectors $\overleftrightarrow{EFG}$ and $\overleftrightarrow{PQR}$ intersect each other at O. O is the centre of circle.

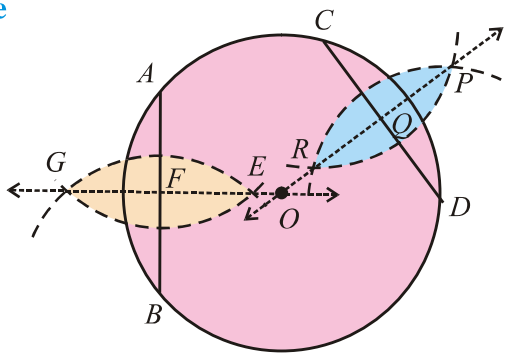


Fig. 13.1.1

13.1(ii)Draw a circle passing through three given non-collinear points:

Given: Three non-collinear points A, B and C.

Steps of Construction:

1. Join A with B and B with C.
2. Draw $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ right bisectors of $\overline{AB}$ and $\overline{BC}$ respectively. $\overleftrightarrow{LM}'$ and $\overleftrightarrow{PQ}'$ intersect at point O.
3. Draw a circle with radius $\overline{OA} = \overline{OB} = \overline{OC}$ having centre at O, which is the required circle.

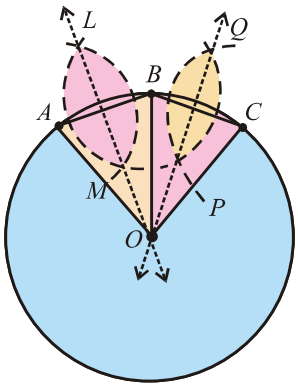


Fig. 13.1.2

13.1(iii-a) To complete the circle by finding the centre when a part of a circumference is given

Given: $\overset{\frown}{AB}$ is Part of circumference of a circle

Steps of Construction:

1. Let C, D, E and F be the four points on the given arc AB.
2. Draw chord $\overline{CD}$ and $\overline{EF}$.
3. Draw $\overleftrightarrow{PQ}$ as perpendicular bisector of $\overline{CD}$ and $\overleftrightarrow{LM}$ as perpendicular bisector of $\overline{EF}$.
4. $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ intersect at O.
   $\therefore$ O is equidistant from points A, B, C, D, E and F.
5. Complete the circle with centre O and radius ($\overline{OA} = \overline{OB} = \overline{OC} = \overline{OD} = \overline{OE} = \overline{OF}$). This will pass through all the points A, B, C, D, E and F on the given part of the circumference.

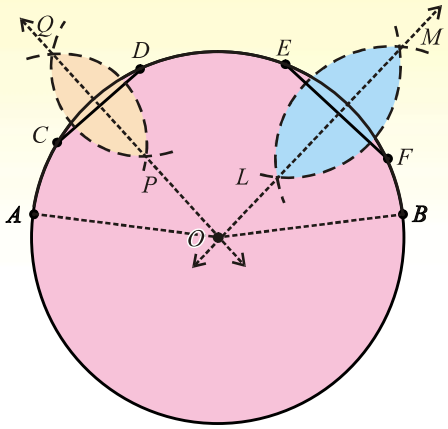


Fig. 13.1.3

13.1(iii-b) To complete the circle without finding the centre when a part of its circumference is given

Given: $\overset{\frown}{AB}$ is the part of circumference of a circle

Steps of Construction:

1. Take two chords $\overline{CD}$ and $\overline{DE}$ of the suitable same length such that these are chords of $\overset{\frown}{AB}$.
2. Produce $\overline{CD}$ to D' and $\overline{DE}$ to E' such that to get the external angle $D'DE'$.
3. Construct $\angle E'EF \cong \angle D'DE'$ and take $m\overline{EF} = m\overline{CD} = m\overline{DE}$. Produce $\overline{EF}$ to F'.
4. Construct $\angle F'FG \cong \angle E'EF'$ and take $m\overline{FG} = m\overline{CD}$. Produce $\overline{FG}$ to G'.

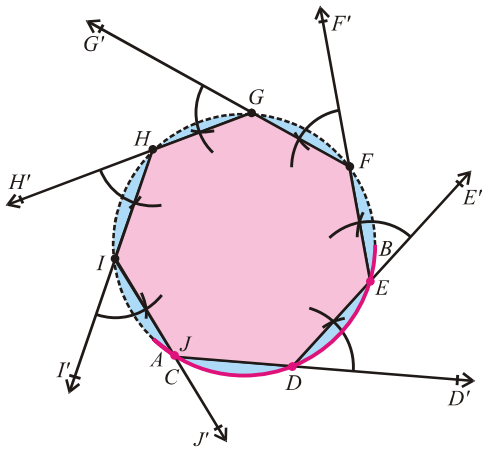


Fig. 13.1.4

5. Points F and G are on the circumference of the required circle. The dotted arcs $\overset{\frown}{EF}$ and $\overset{\frown}{FG}$ are shown in the figure.
6. Continue this process of external angles of equal measure to complete the circumference of the circle as shown in the figure.

Note: Constructing internal angles of equal measure, the circumference of the circle can also be completed.

EXERCISE 13.1

1. Divide an arc of any length
   (i) into two equal parts.
   (ii) into four equal parts.
2. Practically find the centre of an arc ABC.
3. (i) If $|\overline{AB}| = 3$ cm and $|\overline{BC}| = 4$ cm are the lengths of two chords of an arc, then locate the centre of the arc.
   (ii) If $|\overline{AB}| = 3.5$ cm and $|\overline{BC}| = 5$ cm are the lengths of two chords of an arc, then locate the centre of the arc.
4. For an arc draw two perpendicular bisectors of the chords $\overline{PQ}$ and $\overline{QR}$ of this arc, construct a circle through P, Q and R.
5. Describe a circle of radius 5 cm passing through points A and B, 6 cm apart. Also find distance from the centre to the line segment AB.
6. If $|\overline{AB}| = 4$ cm and $|\overline{BC}| = 6$ cm, such that $\overline{AB}$ is perpendicular to $\overline{BC}$, construct a circle through points A, B and C. Also measure its radius.

13.2 CIRCLES ATTACHED TO POLYGONS

13.2(i) Circumscribe a circle about a given triangle.

Given: Triangle ABC.

Steps of Construction:

1. Draw $\overleftrightarrow{LMN}$ as perpendicular bisector of side $\overline{AB}$.
2. Draw $\overleftrightarrow{PQR}$ as perpendicular bisector of side $\overline{AC}$.
3. $\overleftrightarrow{LN}$ and $\overleftrightarrow{PR}$ intersect at point O.
4. With centre O and radius

   $m\overline{OA} = m\overline{OB} = m\overline{OC}$, draw a circle.

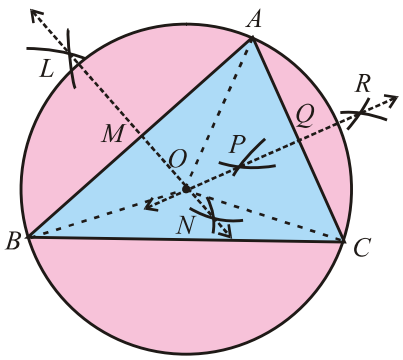


Fig. 13.2.1

This circle will pass through A, B and C whereas O is the circumcentre of the circumscribed circle.

Remember: The circle passing through the vertices of triangle ABC is known as circumcircle, its radius as circumradius and centre as circumcentre.

13.2(ii)Inscribe a circle in a given triangle:

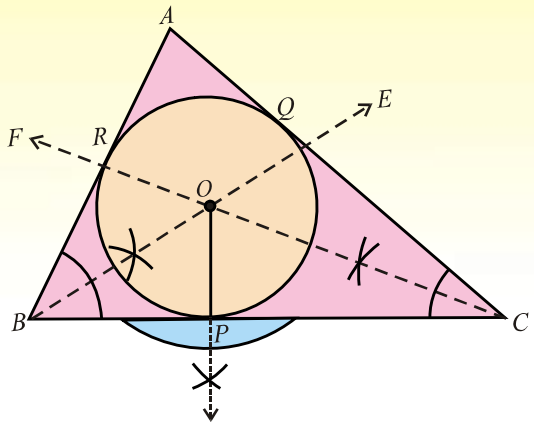


Fig. 13.2.2

Given: A triangle ABC.

Steps of Construction:

1. Draw $\overrightarrow{BE}$ and $\overrightarrow{CF}$ to bisect the angles ABC and ACB respectively. Rays $\overrightarrow{BE}$ and $\overrightarrow{CF}$ intersect each other at point O.
2. O is the centre of the inscribed circle.
3. From O draw $\overrightarrow{OP}$ perpendicular to $\overline{BC}$.

   With centre O and radius $\overline{OP}$ draw a circle. This circle is the inscribed circle of triangle ABC.

Remember:

A circle which touches the three sides of a triangle internally is known as incircle, its radius as in-radius and centre as in-centre.

13.2(iii) Escribe a circle to a given triangle:

Given: A triangle ABC

Steps of Construction:

1. Produce the sides $\overline{AB}$ and $\overline{AC}$ of ΔABC.
2. Draw bisectors of exterior angles ABC and ACB. These bisectors of exterior angles meet at I_1.
3. From I_1 draw perpendicular on side $\overline{BC}$ of ΔABC. Which $\overline{I_1D}$ intersect $\overline{BC}$ at D. I_1D is the radius of the escribed circle with centre at I_1.
4. Draw the circle with radius $\overline{I_1D}$ and centre at I_1 that will touch the side BC of the ΔABC externally and the produced sides AB and AC.

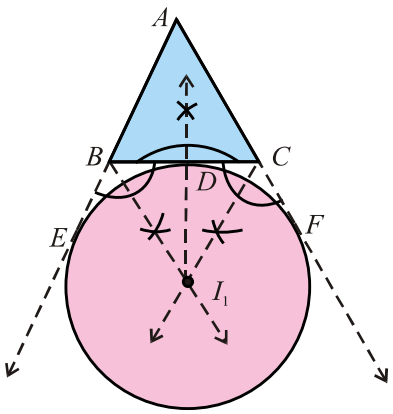


Fig. 13.2.3

Escribed circle: The circle touching one side of the triangle externally and two produced sides internally is called escribed circle (e-circle). The centre of e-circle is called e-centre and radius is called e-radius.

13.2(iv) Circumscribe an equilateral triangle about a given circle

Given: A circle with centre O of reasonable radius.

Steps of Construction:

1. Draw $\overline{AB}$, the diameter of the circle for locating.
2. Draw an arc of radius m $\overline{OA}$ with centre at A for locating points C and D on the circle.
3. Join O to the points C and D.
4. Draw tangents to the circle at points B, C and D.
5. These tangents intersect at points E, F and G.

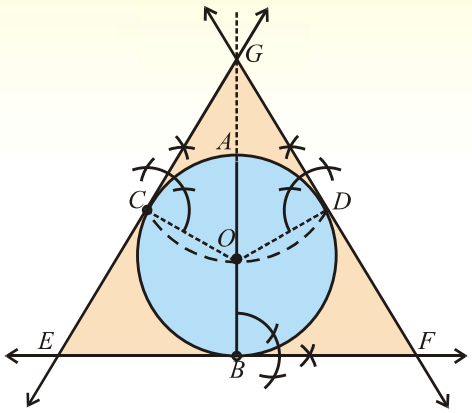


Fig. 13.2.4

13.2(v) Inscribe an equilateral triangle in a given circle.

Given: A circle with centre at O.

Steps of Construction:

1. Draw any diameter $\overline{AB}$ of the circle.
2. Draw an arc of radius OA from point A. The arc cuts the circle at points C and D.
3. Join the points B, C and D to form straight line segments $\overline{BC}$, $\overline{CD}$ and $\overline{BD}$.

Triangle BCD is the required inscribed equilateral triangle.

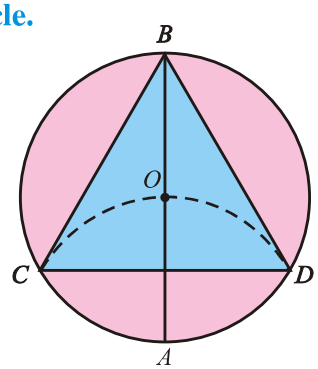


Fig. 13.2.5

13.2(vi) Circumscribe a square about a given circle.

Given: A circle with centre at O.

Steps of Construction:

1. Draw two diameters $\overline{PR}$ and $\overline{QS}$ which bisect each other at right angle.
2. At points P, Q, R and S draw tangents to the circle.
3. Produce the tangents to meet each other at A, B, C and D. $ABCD$ is the required circumscribed square.

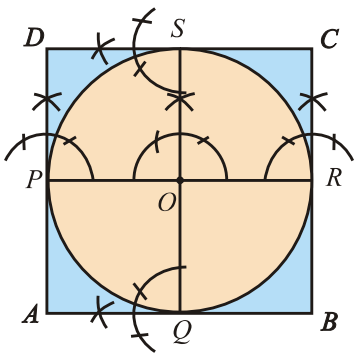


Fig. 13.2.6

13.2(vii) Inscribe a square in a given circle

Given: A circle, with centre at O.

Steps of Construction:

1. Through O draw two diameters $\overline{AC}$ and $\overline{BD}$ which bisect each other at right angle.
2. Join A with B, B with C, C with D, and D with A.

$ABCD$ is the required square inscribed in the circle.

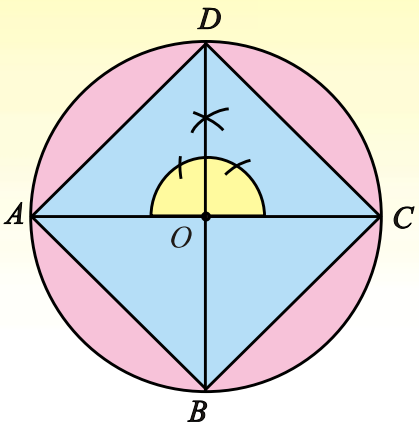


Fig. 13.2.7

13.2(viii) Circumscribe a regular hexagon about a given circle.

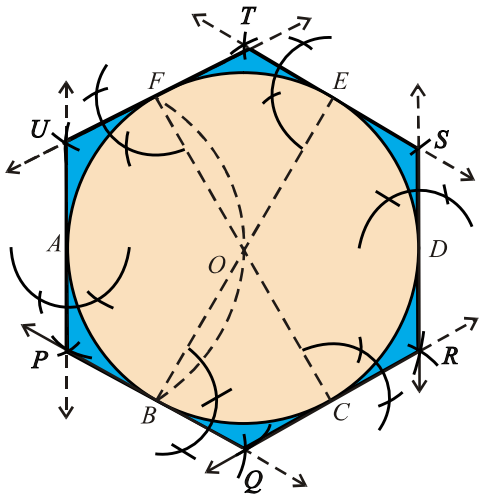


Fig. 13.2.8

Given: A circle with centre at O.

Steps of Construction:

1. Draw any diameter $\overline{AD}$.
2. From point A draw an arc of radius $\overline{AO}$ (the radius of the circle), which cuts the circle at points B and F.
3. Join B with O and extend it to meet the circle at E.
4. Join F with O and extend it to meet the circle at C.
5. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U respectively.

Thus $PQRSTU$ is the circumscribed regular hexagon.

13.2(ix) Inscribe a regular hexagon in a given circle:

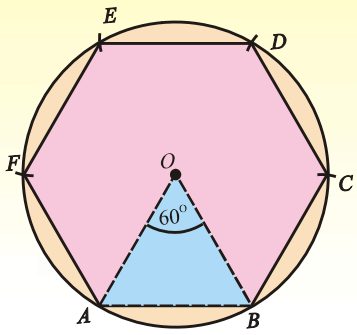


Fig. 13.2.9(a)

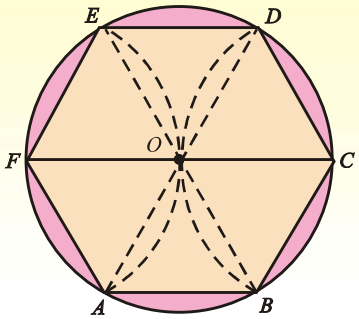


Fig. 13.2.9(b)

Given: A circle, with centre at O.

Steps of Construction:

1. Take any point A on the circle and point with O.
2. From point A, draw an arc of radius $\overline{OA}$ which intersects the circle at point B and F.
3. Join O and A with points B and F.
4. ΔOAB and ΔOAF are equilateral triangles therefore $\angle AOB$ and $\angle AOF$ are of measure 60^o *i.e.*, $m\overline{OA} = m\overline{AB} = m\overline{AF}$.
5. Produce $\overline{FO}$ to meet the circle at C. Join B to C. Since in $\angle BOC = 60$ therefore $m\overline{BC} = m\overline{OA}$.
6. From C and F, draw arcs of radius $\overline{OA}$, which intersect the circle at points D and E.
7. Join C to D, D to E and E to F ultimately. We have
   $$m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF}$$
   Thus the figure $ABCDEF$ is a regular hexagon inscribed in the circle.

EXERCISE 13.2

1. Circumscribe a circle about a triangle ABC with sides
   $|AB| = 6$ cm , $|BC| = 3$cm , $|CA| = 4$cm
   Also measure its circum radius.
2. Inscribe a circle in a triangle ABC with sides
   $|AB| = 5$ cm, $|BC| = 3$ cm, $|CA| = 3$ cm. Also measure its in-radius.
3. Escribe a circle opposite to vertex A to a triangle ABC with sides
   $|AB| = 6$ cm, $|BC| = 4$ cm, $|CA| = 3$ cm. Find its radius also.
4. Circumscribe a circle about an equilateral triangle ABC with each side of length 4cm.
5. Inscribe a circle in an equilateral triangle ABC with each side of length 5cm.
6. Circumscribe and inscribe circles with regard to a right angle triangle with sides, 3cm, 4cm and 5cm.
7. In and around the circle of radius 4cm draw a square.
8. In and around the circle of radius 3.5cm draw a regular hexagon.
9. Circumscribe a regular hexagon about a circle of radius 3cm.

13.3 TANGENT TO THE CIRCLE

13.3(i) To draw a tangent to a given arc without using the centre through a given point *P*:

Case (i) When *P* is the middle point of the arc

Given: *P* is the mid-point of an arc *AB*.

Steps of Construction:

1. Join *A* and *B*, to form the chord $\overline{AB}$.
2. Draw the perpendicular bisector of chord $\overline{AB}$ which passes through mid point *P* of $\overset{\frown}{AB}$ and mid point *R* of $\overline{AB}$.
3. At points *P* construct a right angle *TPR*.
4. Produce $\overrightarrow{PT}$ in the direction of *P* beyond point *S*. Thus $\overleftrightarrow{TP}$ is the required tangent to the arc *AB* at point *P*.

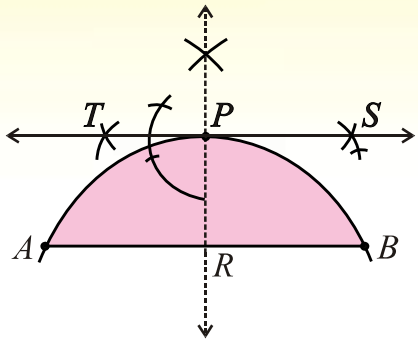


Fig. 13.3.1(*a*)

Case (ii) When *P* is at end point of the arc

Given: *P* is the end point of arc *PQR*.

Steps of Construction:

1. Take a point *A* on the arc *PQR*.
2. Join the points *A* and *P*.
3. Draw perpendicular $\overrightarrow{AS}$ at *A* which intersects the arc *PQR* at *B*.
4. Join the points *B* and *P*.
5. Draw $\angle APD$ of measure equal to that of $\angle ABP$.
6. Now $m\angle BPD = m\angle BPA + m\angle APD$
   $= m\angle BPA + m\angle ABP$
   $= 90^{o}$

Thus $\overleftrightarrow{PD}$ is the required tangent.

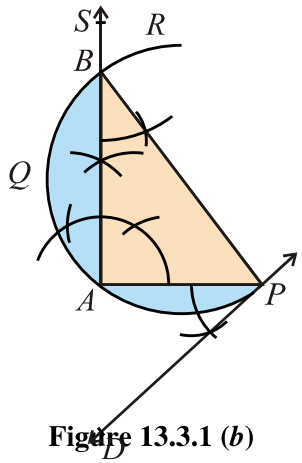


Figure 13.3.1 (*b*)

[$\because$ $m\angle APD = m\angle ABP$]

Case (iii): When point *P* is outside the arc.

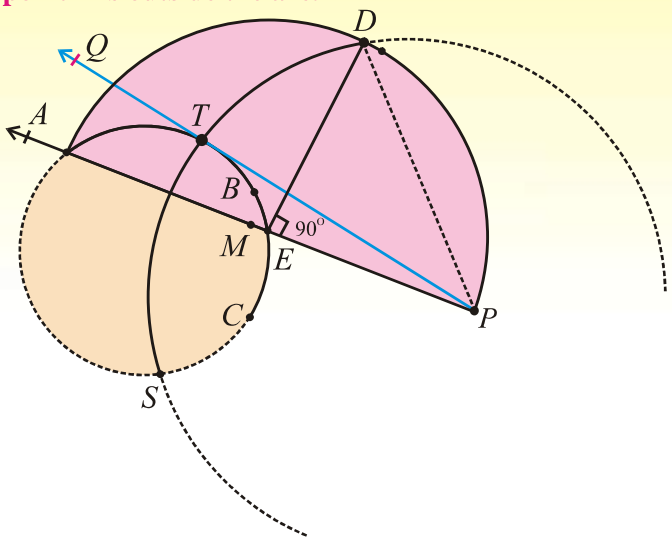


Fig. 13.3.1 (*c*)

Given: Point *P* is a line segmetn out side the arc *ABC* without knowing its centre.

Steps of Construction:

1. Join *A* to *P*. $\overline{AP}$ cuts the arc at *E*.
2. Find mid-point *M* of *AP*.
3. Draw a semi circle of radius |*AM*| = |*MP*| with center at *M*.
4. Draw perpendicular at point *E* which meets the semi circle at *D*.
5. Draw an arc of radius |$\overline{PD}$| with *P* as its center.
6. This arc cuts the given arc *ABC* at points *T*.
7. Join *P* with *T*.

$\overrightarrow{PTQ}$ is the required tangent.

13.3(ii-a) To draw a tangent to a circle from a given point *P* at a given point on the circumference:

Given: A circle with the centre *O* and some point *P* lies on the circumference.

Steps of Construction:

1. Join point *P* to the centre *O*, so that $\overline{OP}$ is the radius of the circle.
2. Draw a line *TPS* which is perpendicular to the radius $\overline{OP}$.

∴ $\overleftrightarrow{TPS}$ is the required tangent to the circle at given point *P*.

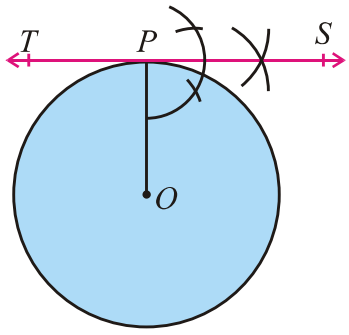


Fig. 13.3.2(*a*)

13.3(ii-b) To draw a tangent to a circle from a given point P which lies outside the circle:

Given: A circle with centre O and some point P out side the circle.

Steps of Construction:

1. Join point P to the centre O.
2. Find M, the mid point of $\overline{OP}$.
3. Construct a semi circle on diameter $\overline{OP}$, with M as its centre. This semi circle cuts the given circle at T.
4. Join P with T and produce $\overline{PT}$ on both sides, then $\overleftrightarrow{PT}$ is the required tangent.

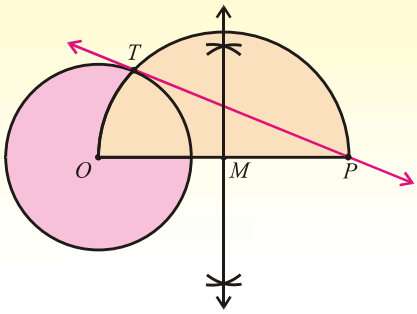


Fig. 13.3.2 (*b*)

13.3(iii) To draw two tangents to a circle meeting each other at a given angle:

Given: A circle with centre O, $\angle MNS$ is a given angle.

Steps of Construction:

1. Take a point A on the circumference of circle having centre O.
2. Join the points O and A.
3. Draw $\angle COA$ of measure equal to that of $\angle MNS$.
4. Produce $\overline{CO}$ to meet the circle at B.
5. $m\angle AOB = 180^o - m\angle COA$
6. Draw $\overleftrightarrow{AD}$ perpendicular to $\overline{OA}$.
7. Draw $\overleftrightarrow{BE}$ perpendicular to $\overline{OB}$.
8. $\overleftrightarrow{AD}$ and $\overleftrightarrow{BE}$ intersect at P.
9. $m\angle AOB + m\angle APB = 180^o$, that is, $m\angle AOB = 180^o - m\angle APB$
10. From step 5 and step 9, we have

$$180^o - m\angle COA = 180^o - m\angle APB \Rightarrow m\angle COA = m\angle APB$$

$$\Rightarrow \quad m\angle APB = m\angle MNS \qquad (\because m\angle COA = m\angle MNS)$$

11. $\overleftrightarrow{AP}$ and $\overleftrightarrow{BP}$ are the required tangents meeting at the given $\angle MNS$.

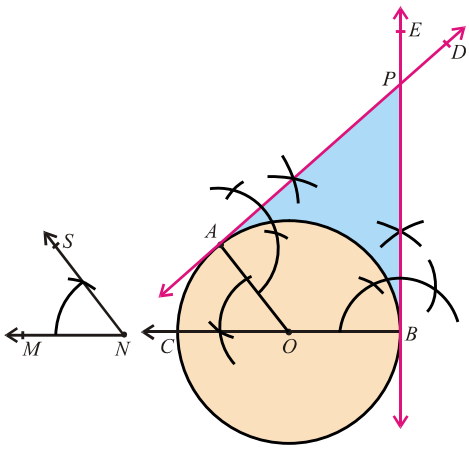


Fig. 13.3.3

13.3(iv-a) To draw direct or (external) common tangents to equal circles:

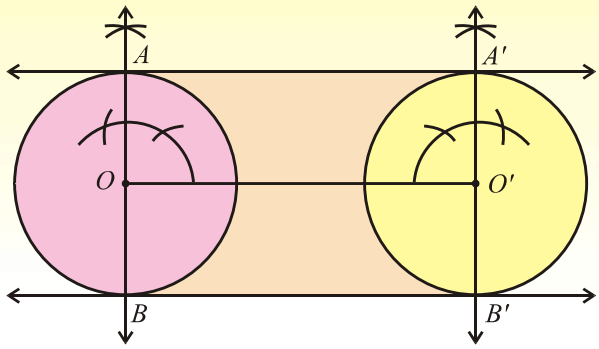


Fig. 13.3.4 (*a*)

Given: Two circles of equal radii with centres O and O' respectively.

Steps of Construction:

1. Join the centres O and O'.
2. Draw diameter AOB of the first circle so that $\overline{AOB} \perp \overline{OO'}$.
3. Draw diameter $A'O'B'$ of the second circle so that $\overline{A'O'B'} \perp \overline{OO'}$.
4. Draw $\overleftrightarrow{AA'}$ and $\overleftrightarrow{BB'}$ which are the required common tangents.

13.3(iv-b) To draw transverse or (internal) common tangents to two equal circles:

Given: Two equal circles with centres O and O' respectively.

Steps of Construction:

1. Join the centres O and O'.
2. Find mid-point M of $\overline{OO'}$.
3. Find mid-point N of $\overline{MO'}$.
4. Taking point N as centre and radius equal to $m\overline{MN}$, draw a circle intersecting the circle with centre O' at points P and P'.
5. Draw a line through the points M and P touching the second circle at the point Q.
6. Draw a line through the points M and P' touching the second circle at the point Q'. Thus $\overleftrightarrow{PQ}$ and $\overleftrightarrow{P'Q'}$ are the required transverse common tangents to the given circles.

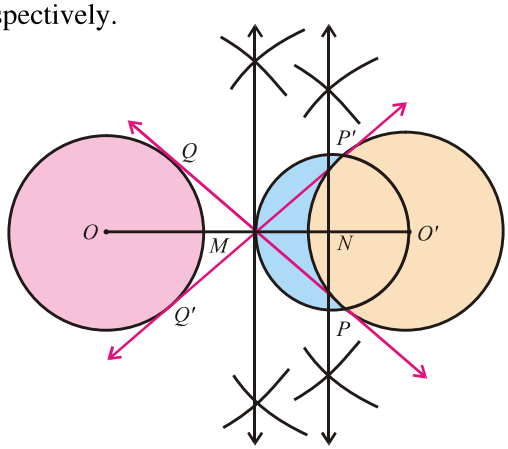


Fig. 13.3.4 (*b*)

13.3(v-a) To Draw direct or (external) common tangents to (two) unequal circles:

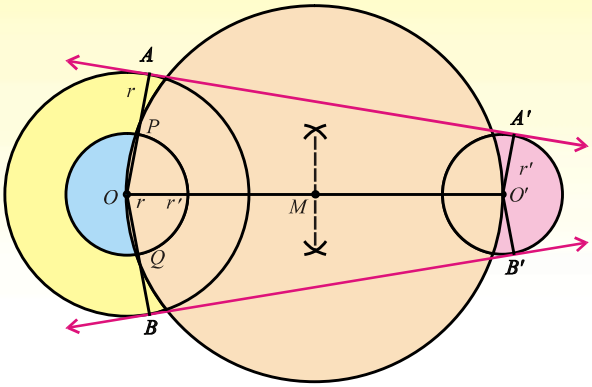


Fig. 13.3.5 (*a*)

Given: Two unequal circles with centres O, O' and radii $r, r'(r > r')$ respectively.

Steps of Construction:

1. Join the points O and O'.
2. On diameter $\overline{OO'}$, construct a new circle with centre M, the mid-point of $\overline{OO'}$.
3. Draw another circle with centre at O and radius = $r - r'$, cutting the circle with diameter $\overline{OO'}$ at P and Q.
4. Produce $\overline{OP}$ and $\overline{OQ}$ to meet the first circle at A and B respectively.
5. Draw $\overrightarrow{O'A'} \parallel \overline{OA}$ and $\overrightarrow{O'B'} \parallel \overline{OB}$.
6. Join AA' and BB' which are the required direct common tangents.

   Thus $\overleftrightarrow{AA'}$ and $\overleftrightarrow{BB'}$ are the required common tangents.

13.3(v-b) To draw to transverse or internal common tangents to two unequal circles:

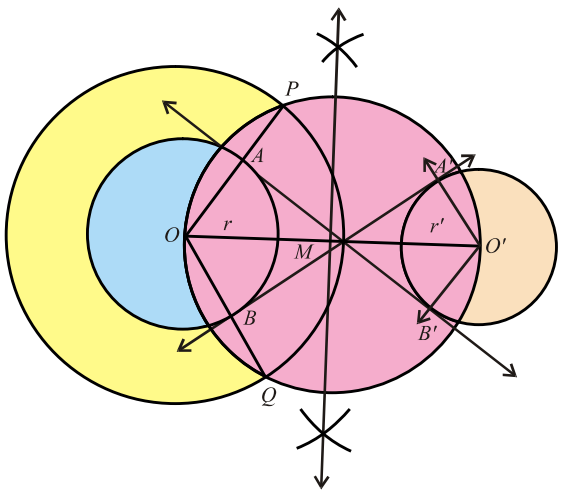


Fig. 13.3.5 (*b*)

Given: Two unequal circles with centres O, O' and radii r, r' respectively.

Steps of Construction:

1. Join the centres O and O' of the given circles.
2. Find the mid point M of $\overline{OO'}$.
3. On diameter OO', construct a new circle with centre M.
4. Draw an other circle with centre at O and radius = $r + r'$ intersecting the circle of diameter OO' at P and Q.
5. Join O with P and Q. $\overline{OP}$ and $\overline{OQ}$ meet the circle with radius r at A and B respectively.
6. Draw $\overrightarrow{OB'} \parallel \overline{OA}$ and $\overrightarrow{OA'} \parallel \overline{OB}$.
7. Join A with B' and A' with B. Thus $\overleftrightarrow{AB'}$ and $\overleftrightarrow{A'B}$ are the required transverse common tangents.

13.3(vi-a) To draw a tangent to two unequal touching circles:

Case I:

Given: Two unequal touching circles with centres O and O'.

Steps of Construction:

1. Join O with O' and produce $\overline{OO'}$ to meet the circles at the point A where these circles touch each other. Fig. 1
2. Tangent is perpendicular to the line segment $\overline{OA}$.
3. Draw perpendicular to $\overline{OA}$ at the point A which is the required tangent.

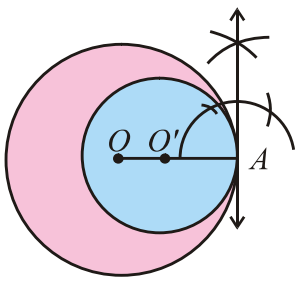


Fig. case-I

Case II:

Given: Two unequal touching circles with centres O and O'.

Steps of Construction:

1. Join O with O'. $\overline{OO'}$ intersects the circles at the point B where these circles touch each other. See Fig. 2.
2. Tangent is perpendicular to line segment containing the centres of the circles.
3. Draw perpendicular to $\overline{OO'}$ at the point B which is the required tangent.

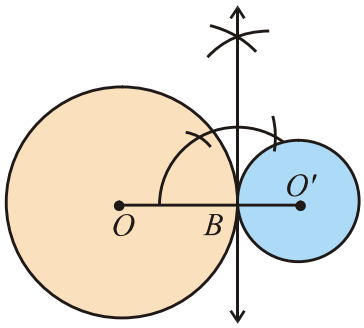


Fig. case-II

Fig. 13.3.6 (*a*)

13.3(vi-b) To draw a tangent to two unequal intersecting circles:

Given: Two intersecting circles with centres A and B.

Steps of Construction:

1. Take a line segment $\overline{AB}$.
2. Draw two circles of radii r and r_1 (where $r > r_1$) with centres at A and B respectively.
3. Taking centre at A, draw a circle of radius $r - r_1$.
4. Bisect the line segment AB at point M.
5. Taking centre at M and radius = $m\overline{AM}$ = $m\overline{BM}$, draw a circle intersecting the circle of radius $r - r_1$ at P and Q.
6. Join the point A with P and produce it to meet the circle with centre A at D. Also join A with Q and produce it to meet the circle with centre A at C.
7. Draw $\overrightarrow{BN}$ parallel to $\overline{AD}$, intersecting the circle with centre B at T.

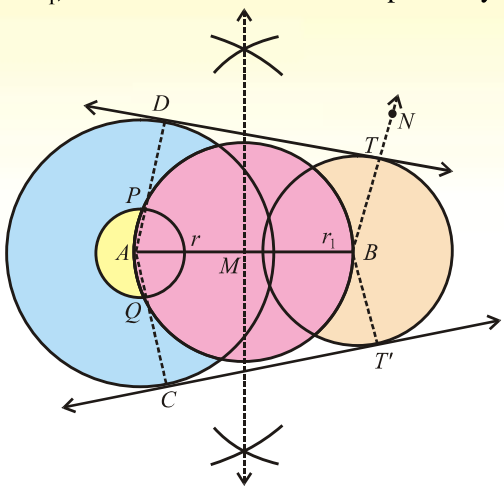


Fig. 13.3.6 (*b*)

8. Draw a line joining the points D and T. $\overleftrightarrow{DT}$ is a common tangent to the given two circles.
9. Repeat the same process on the other side of $\overline{AB}$. $\overleftrightarrow{CT'}$ is also a common tangent to the given two circles.

13.3(vii-a) To draw a circle which touches both the arms of a given angle:

Given: An angle $\angle BAC$.

Steps of Construction:

1. Draw $\overrightarrow{AD}$ bisecting $\angle BAC$.
2. Take any point E on $\overrightarrow{AD}$.
3. Draw $\overrightarrow{ET}$ perpendicular to $\overrightarrow{AC}$ intersecting $\overrightarrow{AC}$ at the point F.
4. Draw a circle with centre E and radius m$\overline{EF}$.

   This circle touches both the arms of $\angle BAC$.

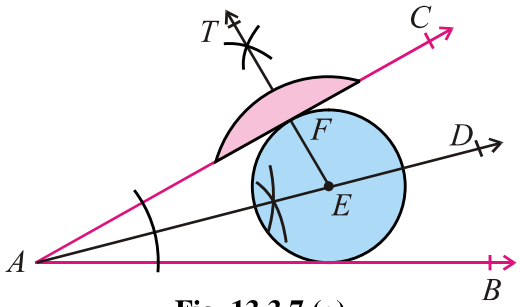


Fig. 13.3.7 (*a*)

13.3(vii-b) To draw a circle touching two convergent lines and passing through a given point between them:

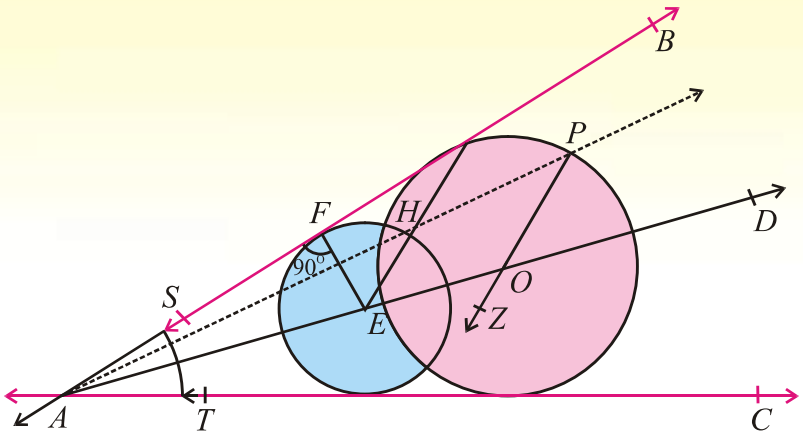


Fig. 13.3.7 (*b*)

Given: $\overleftrightarrow{BS}$ and $\overleftrightarrow{CT}$ are two **converging lines**.

Steps of Construction:

1. Produced $\overleftrightarrow{BS}$ and produced $\overleftrightarrow{CT}$ intersect at A.
2. Draw $\overrightarrow{AD}$ bisecting $\angle BAC$.
3. Take any point E on $\overrightarrow{AD}$.
4. Draw $\overline{EF}$ perpendicular to $\overleftrightarrow{AB}$.
5. Draw a circle with centre E and radius $m\overline{EF}$.
6. This circle touches $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$.
7. Draw $\overrightarrow{AP}$ which cuts this circle at the point H. Join E and H.
8. Through P, draw $\overrightarrow{PZ} \parallel \overline{HE}$ intersecting $\overrightarrow{AD}$ at the point O.
9. Draw a new circle with centre O and radius $m\overline{OP}$. This circle touches both the lines.

13.3(vii-c) To draw a circle which touches three converging lines

Note: It is not possible to draw a circle touching three converging lines.

EXERCISE 13.3

1. In an arc ABC the length of the chord $|BC| = 2$ cm. Draw a secant $|PBC| = 8$ cm, where P is the point outside the arc. Draw a tangent through point P to the arc.
2. Construct a circle with diameter 8 cm. Indicate a point C, 5 cm away from its circumference. Draw a tangent from point C to the circle without using its centre.
3. Construct a circle of radius 2 cm. Draw two tangents making an angle of 60° with each other.
4. Draw two perpendicular tangents to a circle of radius 3 cm.
5. Two equal circles are at 8 cm apart. Draw two direct common tangents of this pair of circles.
6. Draw two equal circles of each radius 2.4 cm. If the distance between their centres is 6 cm, then draw their transverse tangents.
7. Draw two circles with radii 2.5 cm and 3 cm. If their centres are 6.5 cm apart, then draw two direct common tangents.
8. Draw two circles with radii 3.5 cm and 2 cm. If their centres are 6 cm apart, then draw two transverse common tangents.
9. Draw two common tangents to two touching circles of radii 2.5 cm and 3.5 cm.
10. Draw two common tangents to two intersecting circle of radii 3 cm and 4 cm.
11. Draw circles which touches both the arms of angles (i) 45° (ii) 60°.

MISCELLANEOUS EXERCISE - 13

1. Multiple Choice Questions

Three possible answers are given for the following questions. Tick (✓) the correct answer.

(i) The circumference of a circle is called

(*a*) chord (*b*) segment (*c*) boundary

(ii) A line intersecting a circle is called

(*a*) tangent (*b*) secant (*c*) chord

(iii) The portion of a circle between two radii and an arc is called

(*a*) sector (*b*) segment (*c*) chord

(iv) Angle inscribed in a semi-circle is

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{4}$

(v) The length of the diameter of a circle is how many times the radius of the circle

(*a*) 1 (*b*) 2 (*c*) 3

(vi) The tangent and radius of a circle at the point of contact are

(*a*) parallel (*b*) not perpendicular (*c*) perpendicular

(vii) Circles having three points in common

(*a*) over lapping (*b*) collinear (*c*) not coincide

(viii) If two circles touch each other, their centres and point of contact are

(*a*) coincident (*b*) non-collinear (*c*) collinear

(ix) The measure of the external angle of a regular hexagon is

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{\pi}{4}$ (*c*) $\frac{\pi}{6}$

(x) If the incentre and circumcentre of a triangle coincide, the triangle is

(*a*) an isoscenes (*b*) a right triangle (*c*) an equilateral

(xi) The measure of the external angle of a regular octagon is

(*a*) $\frac{\pi}{4}$ (*b*) $\frac{\pi}{6}$ (*c*) $\frac{\pi}{8}$

(xii) Tangents drawn at the end points of the diameter of a circle are

(*a*) parallel (*b*) perpendicular (*c*) Intersecting

(xiii) The lengths of two transverse tangents to a pair of circles are

(*a*) unequal (*b*) equal (*c*) overlapping

(xiv) How many tangents can be drawn from a point outside the circle?

(*a*) 1 (*b*) 2 (*c*) 3

(xv) If the distance between the centers of two circles is equal to the sum of their radii, then the circles will

(*a*) intersect (*b*) do not intersect

(*c*) touch each other externaly

(xvi) If the two circles touches externally, then the distance between their centers is equal to the

(*a*) difference of their radii (*b*) sum of their radii

(*c*) product of their radii

(xvii) How many common tangents can be drawn for two touching circles?

(*a*) 2 (*b*) 3 (*c*) 4

(xviii) How many common tangents can be drawn for two disjoint circles?

(*a*) 2 (*b*) 3 (*c*) 4

2. Write short answers of the following questions

(i) Define and draw the following geometric figures:

(*a*) The segment of a circle. (*b*) The tangent to a circle.

(*c*) The sector of a circle. (*d*) The inscribed circle.

(*e*) The circumscribed circle. (*f*) The escribed circle.

(ii) The length of each side of a regular octagon is 3 cm. Measure its perimeter.

(iii) Write down the formula for finding the angle subtended by the side of a n-sided polygon at the centre of the circle.

(iv) The length of the side of a regular pentagon is 5 cm what is its perimeter?

3. Fill in the blanks

(i) The boundary of a circle is called _____.

(ii) The circumference of a circle is called _____ of the circle.

(iii) The line joining the two points of circle is called _____.

(iv) The point of intersection of perpendicular bisectors of two non-parallel chords of a circle is called the _____.

(v) Circles having three points in common will _____.

(vi) The distance of a point inside the circle from its centre is _____ than the radius.

(vii) The distance of a point outside the circle from its centre is _____ than the radius.

(viii) A circle has only _____ centre.

(ix) One and only one circle can be drawn through three _____ points.

(x) Angle inscribed in a semi-circle is a _____ angle.

(xi) If two circles touch each other, the point of _____ and their _____ are collinear.

(xii) If two circles touch each other, their point of contact and centres are _____.

(xiii) From a point outside the circle _____ tangents can be drawn.

(xiv) A tangent is _____ to the radius of a circle at its point of contact.

(xv) The straight line drawn $\perp$ to the radius of a circle is called the _____ to the circle.

(xvi) Two circles can not cut each other at more than _____ points.

(xvii) The $\perp$-bisector of a chord of a circle passes through the _____.

(xviii) The length of two direct common tangents to two circles are _____ to each other.

(xix) The length of two transverse common tangents to two circles are _____ to each other.

(xx) If the in-centre and circum-centre of a triangle coincide the triangle is _____.

(xxi) Two intersecting circles are not _____.

(xxii) The centre of an inscribed circle is called _____.

(xxiii) The centre of a circumscribed circle is called _____.

(xxiv) The radius of an inscribed circle is called _____.

(xxv) The radius of a circumscribed circle is called _____.

SUMMARY

- A circle of any radius can be traced by rotating a compass about fixed point.
- The perpendicular bisectors of two non-parallel chords of a circle intersect at a point which is known as centre of circle.
- A circle can be drawn through given three non-collinear points.
- When a part of circumference of a circle is given, the circle can be completed.
- If a triangle, the circumscribed circle, inscribed circle and escribed circle opposite to each vertex can be constructed.
- If a circle is given, then the circumscribed and inscribed equilateral triangles can be constructed.
- For a given circle, the circumscribed and inscribed squares can be drawn.

- For a given circle, the circumscribed and inscribed regular hexagon can be constructed.
- We can draw tangents to a given arc as its mid point, its any end point, and a point not on the arc.
- Tangents can be drawn to a given circle, when a point is an its circumference and from a point outside the circle.
- Tangents to two unequal touching circles can be traced.
- Direct or transverse common tangents of two equal circles or two unequal circles can be drawn.
- We can construct a circle touching the arms of a given angle.
- A circle passing through a given point between two converging lines and touching each of them, can be traced.

ANSWERS

Unit 1: Quadratic Equations

EXERCISE 1.1

1. (i) quadratic, $x^2 + 4x - 14 = 0$ (ii) quadratic, $7x^2 - 3x + 7 = 0$
   (iii) quadratic, $4x^2 + 4x - 1 = 0$ (iv) pure, $x^2 - 1 = 0$
   (v) pure, $x^2 - 20 = 0$ (vi) quadratic, $x^2 + 29x + 66 = 0$

2. (i) $\{-4, 5\}$ (ii) $\left\{0, \frac{-5}{2}\right\}$ (iii) $\left\{-2, \frac{2}{17}\right\}$
   (iv) $\{-8, 19\}$ (v) $\{3, -4\}$ (vi) $\left\{\frac{3}{2}, 5\right\}$

3. (i) $\left\{\frac{-1 \pm 2\sqrt{2}}{7}\right\}$ (ii) $\left\{\frac{-2 \pm \sqrt{a^2+4}}{a}\right\}$ (iii) $\left\{3, \frac{1}{11}\right\}$
   (iv) $\left\{\frac{-m \pm \sqrt{m^2 - 4ln}}{2l}\right\}$ (v) $\left\{0, \frac{-7}{3}\right\}$ (vi) $\{-13, 15\}$
   (vii) $\left\{-5, \frac{3}{2}\right\}$ (viii) $\left\{-\frac{1}{2}, -\frac{33}{2}\right\}$ (ix) $\{1,3\}$
   (x) $\{-3a, 4a\}$

EXERCISE 1.2

1. (i) $\left\{\frac{-7 \pm \sqrt{57}}{2}\right\}$ (ii) $\left\{\frac{-4 \pm \sqrt{11}}{5}\right\}$ (iii) $\left\{\sqrt{3}, -\frac{4}{\sqrt{3}}\right\}$
   (iv) $\left\{\frac{3 \pm \sqrt{233}}{8}\right\}$ (v) $\left\{-\frac{1}{3}, \frac{3}{2}\right\}$ (vi) $\left\{\frac{-4 \pm \sqrt{10}}{3}\right\}$
   (vii) $\{3,7\}$ (viii) $\left\{3, \frac{-4}{5}\right\}$
   (ix) $\left\{(a+b), \frac{1}{2}(a+b)\right\}$ (x) $\left\{1, \frac{l+m}{l}\right\}$

EXERCISE 1.3

1. $\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$ 2. $\left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$ 3. $\left\{\frac{16}{625}, 1\right\}$
4. $\{216, 729\}$ 5. $\left\{\frac{3}{5}, 1\right\}$ 6. $\{-1, 0, 1\}$
7. $\{6\}$ 8. $\left\{\pm\frac{5}{4}\right\}$ 9. $\left\{-7a, \frac{a}{7}\right\}$
10. $\{\pm 1, 1 \pm \sqrt{2}\}$ 11. $\left\{1, -2, -\frac{1}{2}\right\}$ 12. $\{-3,0\}$

13. $\{0, -1\}$ 14. $\{2,4\}$ 15. $\{1, 3, 2 \pm \sqrt{33}\}$

16. $\{-4,-2,5,7\}$

EXERCISE 1.4

1. $\left\{-1, -\frac{9}{4}\right\}$ 2. $\left\{1\right\}, \left(\frac{-2}{9}\right.$ Extraneous$\left.\right)$ 3. $\left\{\frac{5}{16}\right\}$, $(-1$ Extraneous$)$

4. $\{7\}$, $(-12$ Extraneous$)$ 5. $\{4\}$ 6. $\{3\}$ 7. ϕ or $\{\ \}$

8. $\{0\}$, $(-3a$ Extraneous$)$ 9. $\left\{\frac{-1 \pm \sqrt{6}}{2}\right\}$ 10. $\left\{\frac{-3 \pm \sqrt{2}}{2}\right\}$ 11. $\{-3, 0\}$

MISCELLANEOUS EXERCISE 1

1. Multiple choice questions:

(i) (b) (ii) (c) (iii) (c) (iv) (a)

(v) (c) (vi) (b) (vii) (a) (viii) (c)

(ix) (a)

2. Short answers:

(i) $-1 \pm \sqrt{3}$ (ii) $0, 3$ (iii) $3x^2 - 2x - 48 = 0$

(iv) (a) Factorization (b) Completing square (c) Quadratic formula

(v) $\frac{-1}{2}, 1$ (vi) $-3, 6$

3. Fill in the blanks:

(a) $ax^2 + bx + c = 0$ (ii) 3

(iii) Completing square (iv) $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(v) $\left\{\pm \frac{1}{5}\right\}$ (vi) Exponential (vii) $\{\pm 3\}$

(viii) Reciprocal (ix) Extraneous (x) Radical sign

Unit 2: Theory Of Quadratic Equations

EXERCISE 2.1

1. (i) 17 (ii) -8 (iii) 0 (iv) 81

2. (i) real, rational and unequal, $x = 8, 15$ (ii) imaginary, $x = \frac{-3 \pm \sqrt{-47}}{4}$

(iii) real and equal, $x = \frac{3}{4}$

(iv) real, irrational and unequal, $x = \frac{-7 \pm \sqrt{205}}{6}$

3. $k = -\frac{1}{3}, 1$ 4. (i) $k = 2, \frac{2}{3}$ (ii) $k = -1, 0$ (iii) $k = 1$

6. $a = mc$

EXERCISE 2.2

1. (i) $-1, -\omega, -\omega^2$ (ii) $2, 2\omega, 2\omega^2$
   (iii) $-3, -3\omega, -3\omega^2$ (iv) $4, 4\omega, 4\omega^2$
2. (i) 128 (ii) 1024 (iii) 125 (iv) 24
   (v) 128 (vi) 2 (vii) -6 (viii) -1

EXERCISE 2.3

1. (i) $S = 5, P = 3$ (ii) $S = -\frac{7}{3}, P = \frac{-11}{3}$
   (iii) $S = \frac{q}{p}, P = \frac{r}{p}$ (iv) $S = \frac{a}{a+b}, P = \frac{b}{a+b}$
   (v) $S = -\frac{m+n}{l+m}, P = \frac{n-l}{l+m}$ (vi) $S = \frac{5m}{7}, P = \frac{9n}{7}$
2. (i) $k = \frac{3}{8}$ (ii) $k = \frac{2}{3}$
3. (i) $k = \frac{64}{23}$ (ii) $k = -1, 2$
4. (i) $p = 0$ (ii) $p = \frac{13}{4}$
5. (i) $m = -55$ (ii) $m = 5$ (iii) $m = -\frac{10}{7}$
6. (i) $m = \frac{3}{2}$ (ii) $m = 1$

EXERCISE 2.4

1. (i) $p^2 - 2q$ (ii) $q(p^2 - 2q)$ (iii) $\frac{1}{q}(p^2 - 2q)$
2. (i) $\frac{5}{6}$ (ii) $\frac{9}{4}$ (iii) $\frac{5}{9}$ (iv) $-\frac{235}{96}$
3. (i) $\frac{-mn^2}{l^3}$ (ii) $\frac{1}{n^2}[m^2 - 2ln]$

EXERCISE 2.5

1. (a) $x^2 - 6x + 5 = 0$ (b) $x^2 - 13x + 36 = 0$
   (c) $x^2 - x - 6 = 0$ (d) $x^2 + 3x = 0$
   (e) $x^2 + 4x - 12 = 0$ (f) $x^2 + 8x + 7 = 0$
   (g) $x^2 - 2x + 2 = 0$ (h) $x^2 - 6x + 7 = 0$
2. (a) $x^2 - 8x + 31 = 0$ (b) $x^2 + 3x + 36 = 0$
   (c) $6x^2 - 3x + 1 = 0$ (d) $2x^2 + x + 2 = 0$
   (e) $2x^2 - 7x + 3 = 0$
3. (a) $x^2 - (p^2 - 2q)\,x + q^2 = 0$ (b) $qx^2 - (p^2 - 2q)\,x + q = 0$

EXERCISE 2.6

1. (i) $Q(x) = x + 6\ ;\ R = -7$ (ii) $Q(x) = 4x^2 - 12x + 31\ ;\ R = -78$
   (iii) $Q(x) = x^2 + 3x + 3\ ;\ R = 8$
2. (i) $h = \frac{7}{3}$ (ii) $h = 6$ (iii) $h = -5$
3. (i) $l = -\frac{3}{2},\ m = -18$ (ii) $l = 2\ ,\ m = -\frac{1}{2}$
4. (i) $-6, 2, 4$ (ii) $-2, \frac{1}{2}, 3$ (iii) $\frac{-3}{4}, -1, 2$
5. (i) $-3, -1, 1, 3$ (ii) $-4, -2, 1, 3$

EXERCISE 2.7

1. $\{(4, 1), (-6, 11)\}$
2. $\{(1, 1), (-5, -8)\}$
3. $\left\{(2, -5), \left(\frac{7}{2}, \frac{-7}{2}\right)\right\}$
4. $\left\{(a, -b), \left(\frac{a-b}{2}, \frac{a-b}{2}\right)\right\}$
5. $\{(-3, 2), (-1, -2)\}$
6. $\{(0, 1), (-3, -2)\}$
7. $\{(\pm 2, \pm 3)\}$
8. $\{(\pm 2, \pm \sqrt{2})\}$
9. $\{(\pm 1, \pm 1)\}$
10. $\left\{\left(\frac{5}{3}, \frac{-1}{3}\right), \left(\frac{-5}{3}, \frac{1}{3}\right), (1, 1), (-1, -1)\right\}$
11. $\left\{(3, 1), (-3, -1), \left(\frac{-4\sqrt{6}}{3}, \sqrt{6}\right), \left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)\right\}$
12. $\left\{\left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right), \left(\frac{-5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}}\right)\right\}$
13. $\left\{\left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(\frac{-7}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right), \left(-\sqrt{3}, \frac{2}{\sqrt{3}}\right), \left(\sqrt{3}, \frac{-2}{\sqrt{3}}\right)\right\}$

EXERCISE 2.8

1. 13, 14 2. 4, 5, 6. 3. 12

4. $\frac{-1}{12}, 2$ 5. $4, -\frac{1}{4}$ 6. 81

7. (3, 6), (6, 3) 8. $x = 5, y = 4$ 9. 11, 7

10. 25cm by 15 cm or 15 cm by 25 cm

MISCELLANEOUS EXERCISE 2

1. Multiple choice questions:

(i) (c) (ii) (b) (iii) (b) (iv) (a)

(v) (a) (vi) (b) (vii) (c) (viii) (c)

(ix) (d) (x) (c) (xi) (a) (xii) (a)

(xiii) (c) (xiv) (d) (xv) (d) (xvi) (a)

2. Short questions:

(i) (*a*) imaginary (*b*) (real) rational, unequal

(c) (real) irrational, unequal (d) (real) rational equal

(ii) $w^2 = \frac{-1 - \sqrt{-3}}{2}$ (iv) 1

(vi) 0 (vii) 64 (viii) $x^2 + 3x + 9 = 0$

(ix) $Q(x) = x^2 + 5x + 10,\ R = 22$ (xi) Sum $= -\frac{3q}{2p}$, Product $= -\frac{2r}{p}$

(xii) $\frac{10}{9}$ (xiii) (a) $\frac{-39}{16}$ (b) $-\frac{13}{8}$ (c) $\frac{\sqrt{-87}}{4}$

(xiv) (a) $x^2 + 5x + 7 = 0$ (b) $x^2 - 10x + 28 = 0$

3. Fill in the blanks:

(i) $b^2 - 4ac$ (ii) equal (iii) real (iv) imaginary

(v) rational (vi) irrational (vii) $-\frac{b}{a}$ (viii) $\frac{c}{a}$

(ix) $\frac{5}{7}$ (x) $\frac{-9}{5}$ (xi) $\frac{1}{\alpha\beta}$ (xii) $1, w, w^2$

(xiii) zero (xiv) w^2 (xv) $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

(xvi) $x^2 + 2x + 4 = 0$

Unit 3: Variations

EXERCISE 3.1

1. (i) $3 : 5 ; \frac{3}{5}$ (ii) $3 : 2 ; \frac{3}{2}$ (iii) $16 : 11 ; \frac{16}{11}$
   (iv) $11 : 24 ; \frac{11}{24}$ (v) $1 : 3 ; \frac{1}{3}$
2. (i) $7 : 12$ (ii) $7 : 5$
3. $4 : 5$ 4. $p = 8$ 5. $x = 1$ 6. $x = 3$; 15 and 24
7. $x = 2$; 8 and 26 8. Rs. 400 9. 51 : 7
10. (i) 7 (ii) $9bx$ (iii) $4l$
11. (i) $x = 2$ (ii) $x = 1$ (iii) $x = 38$
    (iv) $x = p^2 - q^2$ (v) $x = 4$

EXERCISE 3.2

1. (i) $y = 4x$ (ii) $y = 20$ (iii) $x = 7$
2. (i) $y = \frac{7}{3}x$ (ii) $x = 15, y = 42$
3. $R = \frac{5}{8}T$, $R = 40, T = 32$ 4. $R = 32$ 5. $V = \frac{5}{27}R^3$, $R = 15$
6. $w = 3u^3$, $w = 375$ 7. $y = \frac{14}{x}$, $y = \frac{1}{9}$ 8. $y = \frac{12}{x}$, $x = \frac{1}{2}$
9. $w = \frac{35}{z}$, $w = \frac{4}{5}$ 10. $A = \frac{18}{r^2}$, $r = \pm\frac{1}{2}$ 11. $a = \frac{48}{b^2}$, $a = \frac{3}{4}$
12. $V = \frac{135}{r^3}$, $V = \frac{5}{8}$, $r = \frac{3}{4}$ 13. $m = \frac{128}{n^3}$, $m = \frac{16}{27}$, $n = \frac{2}{3}$

EXERCISE 3.3

1. (i) 24 (ii) $9a$ (iii) $\frac{a-b}{a+b}$
   (iv) $(x^2 + xy + y^2)^2$ (v) $(x - 2y)^2$ (vi) $\frac{p-q}{p^2 - pq + q^2}$
2. (i) 24 (ii) $9x^4$ (iii) $14b^2$
   (iv) $5x^3$ (v) $p - q$ (vi) $p^2 - pq + q^2$
3. (i) ± 30 (ii) $\pm 10x^5y^3$ (iii) $\pm 45p^2q^3r^5$
   (iv) $\pm (x - y)$
4. (i) $p = \pm 15$ (ii) $x = \pm 12$ (iii) $p = 8, -4$
   (iv) $m = 17, -11$

EXERCISE 3.4

2. (i) 2 (ii) 2 (iii) $\frac{4(b-a)}{a+b}$ (iv) $\frac{2(z^2-y^2)}{yz}$

(v) 2 (vi) $\left\{\frac{9}{2}, \frac{11}{3}\right\}$ (vii) $\pm\sqrt{\frac{5}{2}}$ (extraneous root), ϕ or $\{\ \}$

(viii) $\{2p, -2p\}$ (ix) $\{7\}$

EXERCISE 3.5

1. $s = \frac{14u^2}{9v}, \frac{28}{5}$ 2. $w = \frac{1}{36}xy^2z, \frac{49}{3}$ 3. $y = \frac{3x^3}{z^2t}, \frac{2}{3}$

4. $u = \frac{7x^2}{4yz^3}, \frac{21}{8}$ 5. $v = \frac{7xy^3}{8z^2}, \frac{14}{3}$ 6. $w = \frac{135}{u^3}, \frac{5}{8}$

EXERCISE 3.7

1. (*i*) $A = 48$ Sq. Units (*ii*) $l = 2$
2. $S = 4\pi r^2$, $r = 3$
3. (*i*) $S = 2.5$ in (*ii*) $F = 16lb$
4. $I = 45$cp 5. $d = 20$ ft 6. Rs. 297000
7. $l = 20$ ft 8. $p = 12$ hp 9. 968000

MISCELLANEOUS EXERCISE 3

1. Multiple choice questions:

(i) (b) (ii) (c) (iii) (b) (iv) (a)
(v) (c) (vi) (a) (vii) (d) (viii) (b)
(ix) (a) (x) (a) (xi) (c) (xii) (b)
(xiii) (a) (xiv) (d) (xv) (a)

2. Short Questions:

(vi) $x = 10$ (vii) $y = \pm\frac{4}{3}$ (viii) $v = 2$

(ix) $\frac{21}{4}$ (x) ± 28 (xi) $\frac{4}{7}$

(xii) $y = \frac{8x^2}{7z}$ (xiii) $z = 6xy$ (xiv) $\frac{18}{v^2}$

3. Fill in the blanks:

(i) $\frac{x+y}{x-y}$ (ii) Antecedent (iii) Consequent

(iv) Extremes (v) Means (vi) $p = 14$

(vii) $m = 8$ (viii) ky (ix) $\frac{v}{k}$

(x) p^2w (xi) $\frac{4}{3}$ (xii) 2

(xiii) $\pm 2mn^2p^3$ (xiv) $m = \pm 6$

Unit 4: Partial Fractions

EXERCISE 4.1

1. $\frac{4}{x+1}+\frac{3}{x-3}$
2. $\frac{-1}{x-4}+\frac{2}{x+3}$
3. $\frac{1}{x-1}+\frac{2}{x+1}$
4. $\frac{-1}{x-1}+\frac{2}{x+3}$
5. $\frac{2}{x-1}+\frac{1}{x+2}$
6. $\frac{3}{x-4}+\frac{4}{x-3}$
7. $1+\frac{9}{5(x-2)}-\frac{4}{5(x+3)}$
8. $2x+3+\frac{5}{3x+1}+\frac{1}{x-1}$

EXERCISE 4.2

1. $\frac{2}{x-1}+\frac{1}{(x-1)^2}-\frac{1}{x-2}$
2. $\frac{2}{x+2}+\frac{1}{(x+2)^2}-\frac{1}{x+3}$
3. $\frac{1}{x-1}-\frac{1}{x+2}-\frac{3}{(x+2)^2}$
4. $x+1-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{x-1}$
5. $\frac{-6}{3x+2}+\frac{2}{x+1}+\frac{3}{(x+1)^2}$
6. $\frac{1}{4(x+1)}-\frac{1}{4(x-1)}+\frac{1}{2(x-1)^2}$
7. $3+\frac{3}{x+2}-\frac{2}{(x+2)^2}$
8. $\frac{1}{4(x-1)}-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}$

EXERCISE 4.3

1. $\frac{-2}{x+3}+\frac{2x-3}{x^2+1}$
2. $\frac{x+12}{5(x^2+1)}-\frac{1}{5(x+3)}$
3. $\frac{1}{2(x+1)}-\frac{x-1}{2(1+x^2)}$
4. $\frac{17x-6}{5(x^2+1)}-\frac{17}{5(x+3)}$
5. $\frac{-2}{13(x+3)}+\frac{2x+33}{13(x^2+4)}$
6. $\frac{1}{2(x+2)}+\frac{x-2}{2(x^2+4)}$
7. $\frac{1}{3(x+1)}-\frac{x-2}{3(x^2-x+1)}$
8. $\frac{2}{3(x+1)}+\frac{x+1}{3(x^2-x+1)}$

EXERCISE 4.4

1. $\frac{x}{x^2+4}-\frac{4x}{(x^2+4)^2}$
2. $\frac{1}{(x+1)}+\frac{x}{(x^2+1)^2}$
3. $\frac{1}{4(1+x)}-\frac{x-1}{4(x^2+1)}+\frac{x-1}{2(x^2+1)^2}$
4. $\frac{1}{4(x-1)}-\frac{x+1}{4(x^2+1)}+\frac{x+1}{2(1+x^2)^2}$

5. $1-\frac{4}{x^2+2}+\frac{4}{(x^2+2)^2}$ 6. $x-\frac{2x}{x^2+1}+\frac{x}{(x^2+1)^2}$

Miscellaneous Exercise 4

1. (i) (*c*) (ii) (*c*) (iii) (*b*) (iv) (*d*) (v) (*c*)
   (vi) (*c*) (vii) (*b*) (viii) (*a*) (ix) (*b*) (x) (*c*)
2. (v) $\frac{-4}{x+2}+\frac{5}{x+3}$ (vi) $\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$
   (vii) $\frac{3}{2}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)$ (viii) $\frac{1}{x-3}+\frac{3}{(x-3)^2}$
   (ix) $\frac{1}{2}\left[\frac{1}{x+a}+\frac{1}{x-a}\right]$ (x) Yes it is an identity.

Unit 5: Sets and Functions

EXERCISE 5.1

1. (i) $\{1, 2, 4, 5, 7, 9\}$ (ii) $\{4, 9\}$ (iii) $\{1, 2, 4, 5, 7, 9\}$
   (iv) $\{4, 9\}$
2. (i) $Y \cup \{13, 17\}$ (ii) $Y \cup \{13, 17\}$ (iii) $\{2, 3, 5, 7, 11\}$
   (iv) $\{2, 3, 5, 7, 11\}$
3. (i) $Y \cup \{13, 17\}$ (ii) T (iii) Y
   (iv) Φ (v) Φ (vi) T
4. (i) $\{18, 20, 21, 22, 24, 25\}$ (ii) $\{18, 20, 21, 22, 24, 25\}$
   (iii) $\{4, 5, \cdots, 10, 12, 14, 15, 16, 18, \cdots, 25\}$
   (iv) $\{4, 5, \cdots, 10, 12, 14, 15, 16, 18, \cdots, 25\}$
5. (i) $\{2, 6, 10, 14, 18\}$ (ii) $\{24\}$
6. (i) Φ (ii) $\{0\}$

EXERCISE 5.2

1. (i) $\{0, 1, 2, 3, \cdots, 20, 23\}$ (ii) $\{0, 1, 2, 3, \cdots, 20, 23\}$ (iii) Φ
   (iv) Φ (v) $\{1, 2, 3, 5, 7, \cdots, 19\}$
   (vi) $\{1, 2, 3, 5, 7, \cdots, 19\}$ (vii) $\{3, 5, 7, 11, 13, 17, 19\}$
   (viii) $\{3, 5, 7, 11, 13, 17, 19\}$

EXERCISE 5.4

1. $A \times B = \{(a, c), (a, d), (b, c), (b, d)\}$
   $B \times A = \{(c, a), (c, b), (d, a), (d, b)\}$
2. $A \times B = \{(0, -1), (0, 3), (2, -1), (2, 3), (4, -1), (4, 3)\}$
   $B \times A = \{(-1, 0), (-1, 2), (-1, 4), (3, 0), (3, 2), (3, 4)\}$

$A \times A = \{(0, 0), (0, 2), (0, 4), (2, 0), (2, 2), (2, 4), (4, 0), (4, 2), (4, 4)\}$

$B \times B = \{(-1, -1), (-1, 3), (3, -1), (3, 3)\}$

3. (i) $a = 6, \quad b = 3$ (ii) $a = 1, \quad b = 7$ (iii) $a = \frac{10}{3}, \quad b = -6$

4. $X = \{a, b, c, d\}; \quad Y = \{a\}$

5. (i) 6 (ii) 6 (iii) 9

EXERCISE 5.5

1. $R_1 = \{(a, 3), (b, 4), (c, 3)\}$
   $R_2 = \{(a, 4), (b, 3), (c, 4)\}$
   $R_3 = \{(3, a), (4, a)\}$
   $R_4 = \{(3, b), (4, b), (3, c), (4, c)\}$

2. $R_1 = \{(-2, -2), (-2, 1), (1, 2), (2, 2)\}$,
   Dom $R_1 = \{-2, 1, 2\} = L$, Range $R_1 = \{-2, 1, 2\}$
   $R_2 = \{(-2, 1), (1, 1), (-2, 2)\}$;
   Dom $R_2 = \{-2, 1\}$, Range $R_2 = \{1, 2\}$

3. $R_1 = \{(a, a), (a, b)$; $R_2 = \{(b, c), (c, c)\}$
   $R_1 = \{(a, d), (b, g)\}$; $R_2 = \{(a, f), (b, e), (c, f)\}$
   $R_1 = \{(d, e), (d, f)\}$; $R_2 = \{(e, e), (f, f), (g, g)\}$

4. $2^{5\times5} = 2^{25}$

5. (i) $R_1 = \{(3, 2), (4, 2), (5, 2), (4, 3), (5, 3)\}$
   (ii) $R_2 = \{(2, 2), (3, 3), (5, 5)\}$
   (iii) $R_3 = \{(1, 5), (3, 3), (4, 2)\}$
   (iv) $R_4 = \{(1, 3), (3, 5), (5, 7)\}$

6. (i) Bijective
   Dom $R_1 = \{1, 2, 3, 4\}$, Range $R_1 = \{1, 2, 3, 4\}$
   (ii) Relation
   Dom $R_2 = \{1, 2, 3\}$, Range $R_2 = \{1, 2, 4, 5\}$
   (iii) Function
   Dom $R_3 = \{b, c, d\}$, Range $R_3 = \{a\}$
   (iv) Onto function
   Dom $R_4 = \{1, 2, 3, 4, 5\}$, Range $R_4 = \{1, 3, 4\}$
   (v) One-one function
   Dom $R_5 = \{a, b, c, d\}$, Range $R_5 = \{a, b, d, e\}$

(vi) Relation

Dom $R_6 = \{1, 2, 3\}$, Range $R_6 = \{2, 3, 4\}$

(vii) One-one function

Dom $R_7 = \{1, 3, 5\}$, Range $R_7 = \{p, r, s\}$

(viii) Relation

Dom $R_8 = \{1, 3, 7\}$, Range $R_8 = \{a, b, c\}$

MISCELLANEOUS EXERCISE 5

1. MCQ's

(i)	(c)	(ii)	(d)	(iii)	(c)	(iv)	(b)	(v)	(d)
(vi)	(c)	(vii)	(d)	(viii)	(c)	(ix)	(b)	(x)	(a)
(xi)	(c)	(xii)	(c)	(xiii)	(a)	(xiv)	(d)	(xv)	(c)
(xvi)	(b)	(xvii)	(b)	(xviii)	(c)	(xix)	(b)	(xx)	(c)

2. Short Questions:

(i) Def. Ex. $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$. A is a subset of B.

(ii) ϕ, $\{a\}$, $\{b\}$, $\{a, b\}$

(x) (i) $(A \cap B)' = A' \cup B'$ (ii) $(A \cup B)' = A' \cap B'$

3. Fill in the Blanks:

(i)	B	(ii)	Disjoint sets	(iii)	$A = B$
(iv)	$(A \cap B) \cup (A \cap C)$	(v)	$(A \cup B) \cap (A \cup C)$		
(vi)	ϕ	(vii)	U	(viii)	ϕ
(ix)	U	(x)	$A \setminus B$	(xi)	IIIrd quadrant
(xii)	IVth quadrant	(xiii)	Zero	(xiv)	Zero
(xv)	$\{a, b, c\}$	(xvi)	$\{a, b, c\}$	(xvii)	John Venn
(xviii)	Binary relation	(xix)	onto	(xx)	not

Unit 6: Basic Statistics

EXERCISE 6.1

4.

Classes	2—3	4—5	6—7	8—9	10—11	12—13	14—15
Frequency	2	1	9	5	6	5	3

a) 6 — 7 b) 4 — 5

EXERCISE 6.2

3. (i) 24.5 (ii) 290

4. (i) 24.5 (ii) 290

5. 32.5

6. A.M = 9.620 G.M=8.553 H.M = 8.089

6. A.M = 9.620 G.M=8.553 H.M = 8.089

7. Mode = 9, 4 Median=7

8. Mode = 2 Median = 2

9. Mean = 10.478 Median = 10.625 Mode = 13.5

10. (i) Weighted Mean = 74 marks (ii) Mean =72.8 marks

11. Weighted Mean = 41.15 rupees per litre

12.

2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
-------	113.33	126	142.66	159.33	178	195.33	208.67	220	-------

EXERCISE 6.3

4. Range = 3500 S.D. ≈ 1417.886

5. a- (i) S.D. = 4.87 (ii) S.D. = 3.87 b- Variance = 6.85

6. Mean = 27.0935 S.D. =3.136

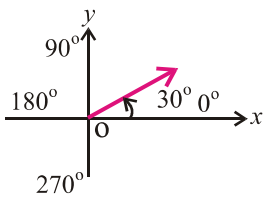
7. Range = 43

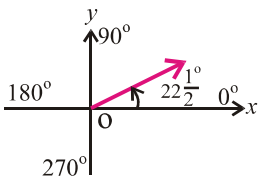
MISCELLANEOUS EXERCISE 6

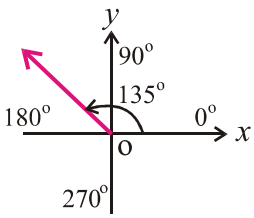
1. (i) (b) (ii) (b) (iii) (a) (iv) (c) (v) (b)
(vi) (a) (vii) (a) (viii) (a) (ix) (b) (x) (c)
(xi) (b) (xii) (a) (xiii) (c) (xiv) (c) (xv) (a)
(xvi) (a) (xvii) (b) (xviii) (b) (xix) (a) (xx) (b)
(xxi) (a) (xxii) (c)

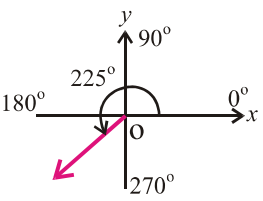
Unit 7: Introduction to Trigonometry

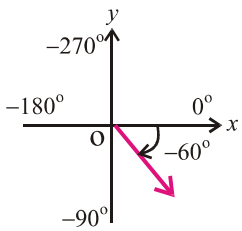
EXERCISE 7.1

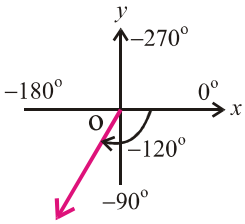
1. (i) 

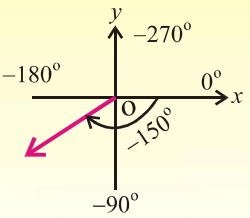
(ii) 

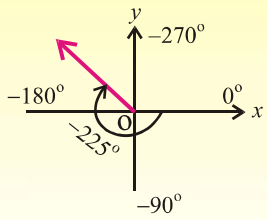
(iii) 

(iv) 

(v) 

(vi) 

(vii) 

(viii) 

2. (i) 45.5° (ii) 60.5083° (iii) 125.3805°

3. (i) $47^\circ 21' 36''$ (ii) $125^\circ 27'$ (iii) $225^\circ 45'$ (iv) $-22^\circ 30'$ (v) $-67^\circ 34' 48''$
(vi) $315^\circ 10' 48''$

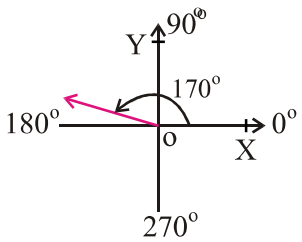
4. (i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{3\pi}{4}$ (iv) $\frac{5\pi}{4}$ (v) $\frac{-5\pi}{6}$
(vi) $\frac{-5\pi}{4}$ (vii) $\frac{5\pi}{3}$ (viii) $\frac{7\pi}{4}$

5. (i) 135° (ii) 150° (iii) 157.5° (iv) 146.25° (v) 171.8869°
(vi) 257.83° (vii) -157.5° (viii) -146.25°

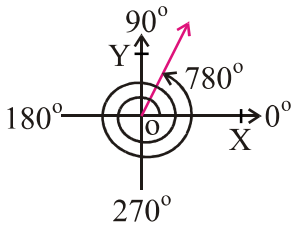
EXERCISE 7.2

1. (i) 0.57rad (ii) 1.8rad 2. (i) 15.4cm (ii) 15.84 mm
3. (i) 16cm (ii) 66.21 cm 4. 18m 5. 220m
6. $\frac{\pi}{2}$ rad 7. 12.57cm 8. 105.56 cm^2
9.(a) 18.85 cm^2 (b) 157.08 cm^2 10. $\frac{49\pi}{18}\ m^2$ or $8.55 m^2$
11. 2972.39 cm^2 12. 31.42 cm^2 13. 5 rad.

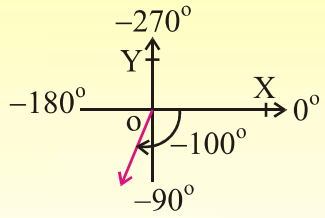
EXERCISE 7.3

1. (i) 

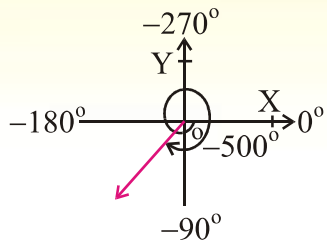
Positive coterminal angle $360^\circ + 170^\circ = 530^\circ$
negative coterminal angle -190°

(ii) 

Positive coterminal angle 60°
negative coterminal angle is -300°

(iii) 

Positive coterminal angle is 260°
negative coterminal angle $-360^\circ - 100^\circ = -460^\circ$

(iv) 

Positive coterminal angle 220°
negative coterminal angle -140°

2. (i) $90^\circ, 180^\circ$ (ii) $270^\circ, 360^\circ$ (iii) $540^\circ, 630^\circ$ (iv) $0^\circ, 90^\circ$

3. (i) $0, \frac{\pi}{2}$ (ii) $\frac{\pi}{2}, \pi$ (iii) $0, \frac{-\pi}{2}$ (iv) $\frac{-\pi}{2}, -\pi$

4. (i) II (ii) III (iii) IV (iv) II (v) I (vi) III

5. (i) *+ve* (ii) *−ve* (iii) *−ve* (iv) *−ve* (v) *+ve* (vi) *−ve*

6. (i) II, $\sin\theta = \frac{3}{\sqrt{13}}$; $\operatorname{cosec}\theta = \frac{\sqrt{13}}{3}$; $\cos\theta = \frac{-2}{\sqrt{13}}$; $\sec\theta = -\frac{\sqrt{13}}{2}$; $\tan\theta = \frac{-3}{2}$; $\cot\theta = \frac{-2}{3}$

(ii) III, $\sin\theta = \frac{-4}{5}$; $\operatorname{cosec}\theta = \frac{-5}{4}$; $\cos\theta = \frac{-3}{5}$; $\sec\theta = \frac{-5}{3}$; $\tan\theta = \frac{4}{3}$; $\cot\theta = \frac{3}{4}$

(iii) I, $\sin\theta = \frac{1}{\sqrt{3}}$; $\operatorname{cosec}\theta = \sqrt{3}$; $\cos\theta = \sqrt{\frac{2}{3}}$; $\sec\theta = \sqrt{\frac{3}{2}}$; $\tan\theta = \frac{1}{\sqrt{2}}$; $\cot\theta = \sqrt{2}$

7. $\sec\theta = \frac{-3}{2}$; $\sin\theta = \frac{\sqrt{5}}{3}$; $\operatorname{cosec}\theta = \frac{3}{\sqrt{5}}$ or $\frac{3\sqrt{5}}{5}$; $\tan\theta = \frac{-\sqrt{5}}{2}$; $\cot\theta = \frac{-2}{\sqrt{5}}$

8. $\sin\theta = \frac{-4}{5}$; $\operatorname{cosec}\theta = \frac{-5}{4}$; $\cos\theta = \frac{-3}{5}$; $\sec\theta = \frac{-5}{3}$; $\cot\theta = \frac{3}{4}$

9. $\tan\theta = -1$; $\sec\theta = \sqrt{2}$; $\operatorname{cosec}\theta = -\sqrt{2}$

10. $\sin\theta = \frac{12}{13}$; $\cos\theta = \frac{5}{13}$; $\sec\theta = \frac{13}{5}$; $\tan\theta = \frac{12}{5}$; $\cot\theta = \frac{5}{12}$

11. (*i*) $\sin\theta = \frac{\sqrt{7}}{4}$; $\operatorname{cosec}\theta = \frac{4}{\sqrt{7}}$; $\cos\theta = \frac{3}{4}$; $\sec\theta = \frac{4}{3}$; $\tan\theta = \frac{\sqrt{7}}{3}$; $\cot\theta = \frac{3}{\sqrt{7}}$

(ii) $\sin\theta = \frac{8}{17}$; $\operatorname{cosec}\theta = \frac{17}{8}$; $\cos\theta = \frac{15}{17}$; $\sec\theta = \frac{17}{15}$; $\tan\theta = \frac{8}{15}$; $\cot\theta = \frac{15}{8}$

(iii) $\sin\theta = \frac{2\sqrt{10}}{7}$; $\operatorname{cosec}\theta = \frac{7}{2\sqrt{10}}$; $\cos\theta = \frac{3}{7}$; $\sec\theta = \frac{7}{3}$; $\tan\theta = \frac{2\sqrt{10}}{3}$; $\cot\theta = \frac{3}{2\sqrt{10}}$

12. (i) $\frac{1}{\sqrt{3}}$ (ii) $\frac{-1}{\sqrt{3}}$ (iii) $\frac{2}{\sqrt{3}}$ (iv) 1 (v) $\frac{-1}{2}$ (vi) $\frac{2}{\sqrt{3}}$ (vii) 0 (viii) 0

(ix) $\frac{-\sqrt{3}}{2}$ (x) $\frac{-1}{2}$ (xi) $\frac{1}{\sqrt{3}}$ (xii) $\frac{-1}{\sqrt{2}}$

EXERCISE 7.4

1. $\tan^2 x$ 2. $\tan^2 x$ 3. $\sin x$ 4. $\sin^2 x$
5. $\tan^2 x$ 6. $\cos^2 x$

EXERCISE 7.5

1. 59.74^o 2. 18.652m 3. 75.5^o or $75^o30'$
4. 27.47^o 5. 4924.04m 6. 3356.4 m 7. 28.72m
8. 0.199 miles 9. 25.94 feet 10. 2928.2 feet 11. 164m ; 164m (or 163.93)
12. 20.33 meter

MISCELLANEOUS EXERCISE 7

Q.1. (i) (*a*) (ii) (*d*) (iii) (*c*) (iv) (*b*) (v) (*c*)
(vi) (*b*) (vii) (*a*) (viii) (*b*) (ix) (*c*) (x) (*b*)

Q.2. (iii) 10800′ (v) 45^o (vi) $\frac{\pi}{12}$ rad. (vii) 2 rad. (viii) 71.27cm (x) $\frac{40}{9}$

Q.3. (i) 180^o (ii) III (iii) IV (iv) $\frac{1}{2}r^2\theta$ (v) 6 cm^2

(vi) $2k\pi + 120^o$ where $k = 1$ (vii) $\theta = 30^o$ or $\frac{\pi}{6}$ rad (viii) 2

(ix) $\text{cosec}^2\theta$ (x) $\frac{1 - \sin\theta}{\cos\theta}$

Unit 8: Projection of a Side of a Triangle

EXERCISE 8.1

1. 2.646 cm , $\frac{\sqrt{3}}{2}$ sq cm 2. $m\overline{AC} = 2\sqrt{29}$ cm

EXERCISE 8.2

1. $m\overline{BC} \approx 5.29$ cm 2. 5.45 cm

MISCELLANEOUS EXERCISE 8

3. $\simeq 4.58$ cm 4. $\simeq 4.12$ cm 5. 15 cm
6. 6 cm 7. 90^o 8. $\simeq (61.9)^0$
9. Acute angled 10. Right angled

Unit 9: Chords of a Circle

EXERCISE 9.1

3. 10 cm 4. $\simeq$ 14.97 cm

EXERCISE 9.2

3. 7 cm

MISCELLANEOUS EXERCISE 9

1. (i) (*c*) (ii) (*a*) (iii) (*d*) (iv) (*c*)
(v) (*a*) (vi) (*b*) (vii) (*c*) (viii) (*b*)
(ix) (*a*) (x) (*c*) (xi) (*b*) (xii) (*b*)
(xiii) (*d*) (xiv) (*c*)

Unit 10: Tangent of a Circle

EXERCISE 10.2

2. 4 cm 3. $\simeq$ 16.96 cm

MISCELLANEOUS EXERCISE 10

1. (i) (*c*) (ii) (*a*) (iii) (*d*) (iv) (*b*)
(v) (*d*) (vi) (c) (vii) (*b*) (viii) (*d*)
(ix) (*c*) (x) (*a*) (xi) (*c*) (xii) (*b*)
(xiii) (*b*)

Unit 11: Chords and Arcs

MISCELLANEOUS EXERCISE 11

1. (i) (*d*) (ii) (*c*) (iii) (*b*) (iv) (*b*)
(v) (*a*) (vi) (*c*) (vii) (*b*) (viii) (*c*)
(ix) (*a*) (x) (*b*)

Unit 12: Angle in a Segment of a Circle

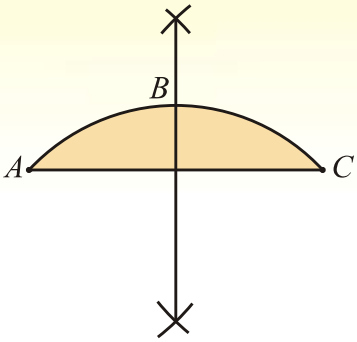
MISCELLANEOUS EXERCISE 12

1. (i) (*c*) (ii) (*d*) (iii) (*a*) (iv) (*c*)
(v) (*b*) (vi) (*d*) (vii) (*d*) (viii) (*b*)
(ix) (*d*) (x) (*c*)

EXERCISE 13.1

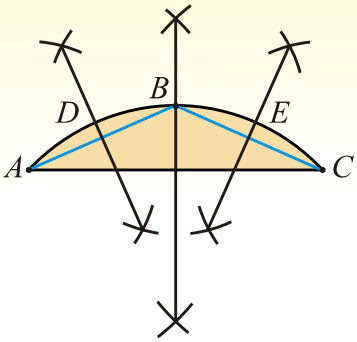
1

(*i*)



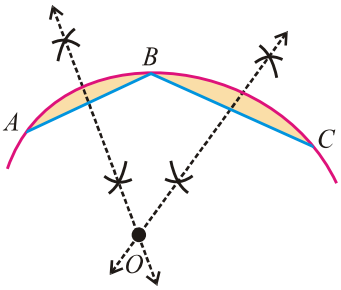
Two equal parts of the arc AC are $\overset{\frown}{AB}$ and $\overset{\frown}{BC}$

(*ii*)

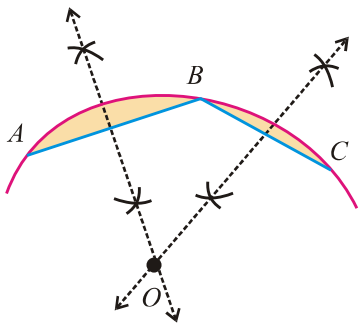


Four equal parts of the arc AC are $\overset{\frown}{AD}$, $\overset{\frown}{DB}$, $\overset{\frown}{BE}$, $\overset{\frown}{EC}$

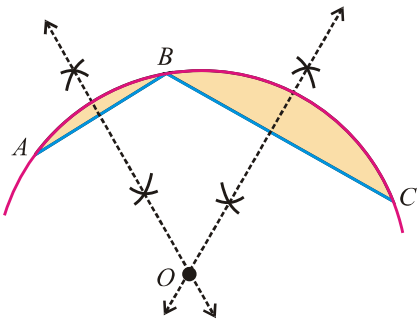
2



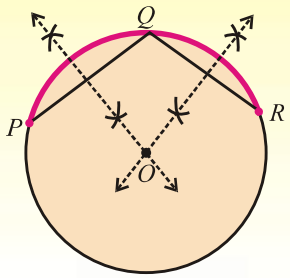
3 (*i*)



(*ii*)



4



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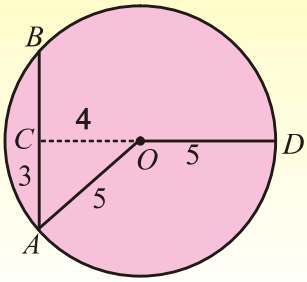
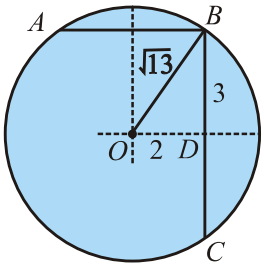


6.



EXERCISE 13.2

1. radius = 3.3 cm. 2. 1 cm (approximately) 3. 2.3 cm

MISCELLANEOUS EXERCISE 13

1. MCQ's

(i)	(*c*)	(ii)	(*b*)	(iii)	(*a*)	(iv)	(*a*)	(v)	(*b*)
(vi)	(*c*)	(vii)	(*a*)	(viii)	(*c*)	(ix)	(*a*)	(x)	(*c*)
(xi)	(*a*)	(xii)	(*a*)	(xiii)	(*b*)	(xiv)	(*b*)	(xv)	(*c*)
(xvi)	(*b*)	(xvii)	(*b*)	(xviii)	(*c*)				

2. (ii) 24 cm (iii) $\frac{360^o}{n}$ (iv) 25 cm

3. Fill in the Blanks:

i.	circumference	ii.	boundary	iii.	chord
iv.	centre	v.	coincide	vi.	less
vii.	greater	viii.	one	ix.	non-collinear
x.	right	xi.	contact, centres	xii.	collinear
xiii.	two	xiv.	perpendicular	xv.	tangent
xvi.	two	xvii.	centre	xviii.	equal
xix.	equal	xx.	equilateral	xxi.	concentric
xxii.	incentre	xxiii.	circumcentre	xxiv.	in-radius
xxv.	circum-radius				

SYMBOLS AND ABBREVIATIONS

Adj. A	Adjoint of A	$\because$	Since or because
A^t	Transpose of A	det A or $\|A\|$	determinant of A
A^{-1}	Inverse of A	π	pi
Add	Addition, adding	$a \times 10^n$	form for scientific notation
$\log_a x$	Logarithm of x to the base a	pt	Point
i	ieota, no. whose square is -1	w.r.t.	With respect to
+ve	positive	−ve	Negative
$\in$	Belongs to	$\notin$	does not belong to
$\forall$	For all	=	Equal to
$\exists$	There exist	$\neq$	Not equal to
Alt	Alternate	$\therefore$	Therefore
Constr	Construction	i.e.	that is
Cor	Corollary	$\Rightarrow$	implies that
Corresp	Corresponding	$^\circ$	degree
Def	Definition	$'$	minute or foot
Ext	Exterior	$''$	second or inch
Fig	Figure	cm	centimeter
Iff	If and only if	$\approx$	nearly equal to
Iso	Isosceles	$\cong$	is congruent to
Mid pt.	Middle point	$\leftrightarrow$	correspondence
perp	Perpendicular	Δ^s	Triangles
prob.	Problem	$\geq$	greater than or equal to
Quad.	Quadrilateral	$\leq$	less than or equal to
Rect	Rectangle	$\underline{\lfloor rt}$	right angle
Rhmb	Rhombus	Δ	Triangle
Sq	Square	$\perp$	is perpendicular to
st line	Straight line	$\|\|$	is parallel to
Th	Theorem	$\|\|$ gm	parallelogram
Trap	Trapezium	$\odot$	circle
vert opp.	Vertically opposite	O^{ce}	circumference
Q.E.D	Quod Erat Demonstramdum	$\overset{\frown}{AB}$	arc AB
θ	Theta (angle measure)	$\overline{AB}$	line segment AB.
ω	Omega	Φ	phi

TABLE OF LOGARITHM

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	9	13	17	21	26	30	34	38
											4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	12	15	19	23	27	31	35
											4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	11	14	18	21	25	28	32
											3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	7	10	13	16	20	23	26	30
											3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	19	22	25	28
											3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	9	11	14	16	20	23	26
											3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	14	17	19	22	24
											3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3	5	8	10	13	15	18	20	23
											2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
											2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
											2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3785	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

TABLE OF LOGARITHM

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8738	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9603	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE OF ANTILOGARITHM

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1027	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	0	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1235	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

TABLE OF ANTILOGARITHM

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	6	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

GLOSSARY

Unit 1

Quadratic equation: An equation which contains the square of the unknown (variable) quantity, but no higher power, is called a *quadratic equation* or an equation of the *second degree.*

Second degree equation

General or standard form A *second degree equation* in one variable x, is $ax^2 + bx + c = 0$, $a \neq 0$ and a, b, c are constants is called the *general or standard form* of a quadratic equation. Where a is the co-efficient of x^2, b is the co-efficient of x and constant term is c.

Reciprocal equation: An equation is said to be a *reciprocal equation,* if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Exponential equations: In *exponential equations* variable occurs in exponent.

Radical equation: An equation involving expression under the *radical sign* is called a *radical equation.*

Unit 2

Discriminant: The expression "$b^2 - 4ac$" of the quadratic expression $ax^2 + bx + c$ is called Discriminant.

Cube roots: The *cube roots* of unity are 1, ω and ω^2.

Complex cube roots: *Complex cube roots* of unity are ω and ω^2.

Properties of cube roots of unity

(a) The *product* of three cube roots of unity is one. *i.e.*, $(1)\ (\omega)\ (\omega^2) = \omega^3 = 1$

(b) Each of the complex cube roots of unity is *reciprocal* of the other.

(c) Each of the complex cube roots of unity is the *square* of the other.

(d) The *sum* of all the cube roots of unity is zero, *i.e.*, $1 + \omega + \omega^2 = 0$

Roots of the quadratic equation: The *roots* of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ are

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum and the product: The *sum* and the *product* of the roots of a quadratic equation

$$\alpha + \beta = \frac{-b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}$$

Symmetric functions: *Symmetric functions* of the roots of a quadratic equation are those functions in which all the roots involved are alike, so that the value of the expression remains unaltered, when roots are interchanged.

Formation of a quadratic equation as:

$x^2 - (\text{sum of the roots})\, x + \text{product of the roots} = 0$

$\Rightarrow \quad x^2 - (\alpha + \beta)\, x + \alpha\beta = 0$

Synthetic division: *Synthetic division* is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial.

Simultaneous equations: A system of equations $f(x, y) = 0$ and $g(x, y) = 0$ having a common solution is called a system of *simultaneous equations*.

Unit 3

Ratio: A relation between two quantities of the same kind is called *ratio*.

Proportion: A *proportion* is a statement, which is expressed as equivalence of two ratios.

If two ratios $a : b$ and $c : d$ are equal, then we can write $a : b = c : d$

Direct variation: If two quantities are related in such a way that when one changes in any ratio so does the other is called *direct variation*.

Inverse variation If two quantities are related in such a way that when one quantity increases, the other decreases is called *inverse variation*.

Theorem on proportions:

(1) *Theorem of Invertendo*

If $a : b = c : d$ then $b : a = d : c$

(2) *Theorem of Alternando*

If $a : b = c : d$, then $a : c = b : d$

(3) *Theorem of Componendo*

If $a : b = c : d$, then

(*i*) $a + b : b = c + d : d$

(*ii*) $a : a + b = c : c + d$

(4) *Theorem of Dividendo*

If $a : b = c : d$, then

(*i*) $a - b : b = c - d : d$

(*ii*) $a : a - b = c : c - d$

(5) *Theorem of Componendo-dividendo*

If $a : b = c : d$, then

$a + b : a - b = c + d : c - d$

Joint variation: A combination of direct and inverse variations of one or more than one variables forms *joint variation*.

K-Method:

If $\frac{a}{b} = \frac{c}{d}$

Then $k = \frac{a}{b} = \frac{c}{d}$ or $a = kb$ and $c = kd$

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ then $a = bk$, $c = dk$ and $e = fk$

Unit 4

Fraction: A *fraction* is an indicated quotient of two numbers or algebraic expressions.

Equation: An *equation* is equality between two expressions.

Identity: An *identity* is an equation which is satisfied by all the values of the variables involved.

Rational fractional: An expression of the form $\frac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are polynomials in x with real coefficient, is called a *rational fractional.* Every fractional expression can be expressed as a quotient of two polynomials.

Proper rational fraction: A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called a *proper rational fraction* if degree of the polynomial $N(x)$, in the numerator is less than the degree of the polynomial $D(x)$, in the denominator.

Improper fraction: A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an *improper fraction* if degree of the polynomial $N(x)$ in the numerator is greater or equal to the degree of the polynomial $D(x)$ in the denominator.

Partial fractions: Decomposition of resultant fraction $\frac{N(x)}{D(x)}$, when

(a) denominator $D(x)$ consists of non-repeated linear factors.

(b) denominator $D(x)$ consists of repeated linear factors.

(c) denominator $D(x)$ contains non-repeated irreducible quadratic factor.

(d) denominator $D(x)$ has repeated quadratic factor.

Unit 5

Set — A set is the *well defined collection* of *distinct* objects with some common properties.

Union of sets — Union of two sets A and B denoted by $A \cup B$ is the set *containing elements* which either belong to A or to B or to both.

Intersection of sets — Intersection of two sets A and B denoted by $A \cap B$ is the set of *common elements* of both A and B. In symbols $A \cap B = \{x : \forall\ x \in A \text{ and } x \in B\}$.

Difference of sets: — The set difference of B and A denoted by $B - A$ is the set of all those elements of B but *do not belonging to* A.

Compliment: — *Complement* of a set A w.r.t. universal set U is denoted by $A^C = A'$ $= U - A$ contains all those elements of U which *do not belong to* A.

Closed figures: — British mathematician John Venn (1834 – 1923) introduced rectangle for a universal set U and its subsets A and B as *closed figures* inside this rectangle.

Specific order: — An ordered pair of elements is written according to a *specific order* for which the order of elements is strictly maintained.

Ordered pairs: — Cartesian product of two non empty sets A and B denoted by $A \times B$ consists of all *ordered pairs* (x, y) such that $\forall\ x \in A$ and $\forall\ y \in B$.

Binary Relation: — Suppose A and B are two non empty sets then *relation* $f : A \to B$ is called a function if (i) Dom f = set A (ii) every $x \in A$ appears in one and only ordered pair $\in f$.

Function: — Suppose A and B are two non empty sets then *relation* $f : A \to B$ is called a function if (i) Dom f = set A (ii) $\forall\ x \in A$ we can associate some unique image element $y = f(x) \in B$.

First elements & second elements: — Dom f is the set consisting of all *first elements* of each ordered pair $\in f$ and range f is the set consisting of all *second elements* of each ordered pair $\in f$.

Into function — A function $f : A \to B$ is called an into function if at least one element in B is not an image of some element of set A *i.e.*, *Range of* $f \subseteq$ *set* B.

Onto function A function $f: A \to B$ is called an onto function if every element of set B is an image of at least one element of set A *i.e.*, *Range of* f = *set* B.

One-one function: A function $f: A \to B$ is called one-one function if all *distinct elements* of A have distinct images in B

Bijective function: A rule $f: A \to B$ is called bijective function iff function f is *one-one and onto*.

Constant function: A function $f: A \to B$ is called a constant function if $\forall\ x \in A$. There is an element $C \in B$ such that $f(x) = C$.

Identity function: A function $f: A \to A$ is called Identity function if $\forall\ x \in A$ we can associate some *unique image element* x itself such that

$$f(x) = x \quad \forall\ x \in A.$$

Unit 6

Frequency distribution: A *frequency distribution* is a tabular arrangement classifying data into different groups.

Class limits

(a) The minimum and the maximum values defined for a class or group are called *class limits*.

(b) The real class limits of a class is called *class boundary*. It is obtained by adding two successive class limits and dividing the sum by 2.

(c) For a given class the average of that class obtained by dividing the sum of upper and lower class limit by 2, is called the *midpoint or class mark* of that class.

(d) The total of frequency up to an upper class limit or boundary is called the *cumulative frequency*.

Histogram A *Histogram* is a graph of adjacent rectangles constructed on XY-plane.

Arithmetic mean *Arithmetic mean* is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their number.

Deviation A *Deviation* is defined as 'a difference of any value of the variable from any constant'. $D_i = x_i - A$.

Geometric mean *Geometric mean* of a variable X is the n^{th} positive root of the product of the $x_1, x_2, x_3, \ldots\ldots, x_n$ observations. In symbols we write,

$$G.M = (x_1.x_2.x_3.....x_n)^{1/n}$$

Harmonic mean *Harmonic mean* refers to the value obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, ..., x_n$ observations.

Mode: *Mode* is defined as the most frequent occurring observation of the variable or data.

$$Mode = L + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

Median: *Median* is the measure which determines the middlemost observation in a data set. $Median = L + \frac{h}{f}\left\{\frac{n}{2} - c\right\}$

Dispersion: Statistically, *Dispersion* means the spread or scatterness of observations in a data set.

Range: *Range* measures the extent of variation between two extreme observations of a data set. It is given by the formula:

$$\text{Range} = X_{max} - X_{min} = X_m - X_0$$

Variance: *Variance* is defined as the mean of the squared deviations of x_i (i = 1, 2, ….., n) observations from their arithmetic mean. In symbols,

$$\text{Variance of } X = \text{Var}(X) = S^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{X}\right)^2}{n} = \frac{\sum\left(X - \bar{X}\right)^2}{n}$$

Standard deviation: *Standard deviation* is defined as the positive square root of mean of the squared deviations of x_i (i = 1, 2, ….., n) observations from their arithmetic mean. In symbols we write,

$$\text{Standard Deviation of } X = \text{S.D}(X) = \sqrt{\frac{\sum\left(X - \bar{X}\right)^2}{n}}$$

Unit 7

Degree: If we divide the circumference of a circle into 360 equal arcs. Then the angle subtended at the centre of the circle by one arc is called one *degree* and is denoted by 1^o.

Radian: The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle, is called one *radian.*

Relationship between radian and degree measure: $1^{o} = \frac{\pi}{180}$ radian , ≈ 0.0175 radian and 1 radian $= \left(\frac{180}{\pi}\right)^{o}$, ≈ 57.295 degrees.

Relation between angle, arc length and radius: Relation between central angle and arc length of a circle: $\boxed{l = r\theta}$

Area of a circular sector: Area of a circular sector, $A = \frac{1}{2} r^2\theta$

Coterminal angle: Two or more than two angles with the same initial and terminal sides are called *coterminal angles.*

Quadrantal angle: An angle is called a *quadrantal angle*, if its terminal side lies on the *x*-axis or *y*-axis.

Standard position: A general angle is said to be in *standard position* if its vertex is at the origin and its initial side is directed along the positive direction of the *x*-axis of a rectangular coordinate system.

Trigonometric ratios: There are six fundamental *trigonometric ratios* (functions) called sine, cosine, tangent, cotangent, secant and cosecant.

Trigonometric Identities: *Trigonometric Identities*(a) $\cos^2\theta + \sin^2\theta = 1$

(b) $\sec^2\theta - \tan^2\theta = 1$

(c) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Unit 8

Projection: The *projection* of a given point on a line is the foot of ⊥ drawn from the point on that line. However the projection of given point *P* on a line *AB* is the point *P* itself.

Zero dimension: The projection of a finite line on an other line is the portion of the latter intercepted between the projection of ends of the given finite line. However projection of a vertical line on an other line is the join of these two intersecting lines which is of *zero dimension*.

Obtuse angle: An angle which is greater than 90^{o} is called *obtuse angle.*

Right angle An angle which is equal to 90^{o} is called *right angle.*

Acute: An angle which is less than 90^{o} is called *acute angle.*

Unit 9

Circle: A *circle* is the locus of a moving point P in a plane which is equidistant from some fixed point N. The fixed paint N not lying on the circle is called the *centre* and the constant distance PN is called its *radius*.

Circumference: $2\pi r$ is the *circumference* of a circle with radius r.

Circular area: πr^2 is the *circular area* of a circle of radius r.

Collinear points: The points lying on the same line are *collinear points* otherwise they are *non-collinear points*.

Circumcirlce: The circle passing through the vertices of a triangle is called its *circumcirlce* where $\perp$ bisectors of sides of the triangle provides the centre.

Unit 10

Secant: A *secant* is a st line which cuts the circumference of a circle in two distinct points.

Tangent: A *tangent* to a circle is the St line which meets the circumference at one point only and being produced does not cut it at all. The point of tangency is also known as the point of contact. AB is the tangent line to the circle C.

Length of a tangent: The *length of a tangent* to a circle is measured from the given point to the point of contact.

Unit 12

Sector: The *sector* of a circle is an area bounded by any two radii and the arc intercepted between them.

Central angle: A *central angle* is subtended by two radii at the centre of the circle.

Circumangle: A *circumangle* is subtended between any two chords of a circle, having common point on its circumference.

Chord: The join of any two points on the circumference of the circle is called its chord.

Cyclic quadrilateral: A quadrilateral is called *cyclic* when a circle can be drawn through its four vertices.

In-centre: *In-centre* of a triangle is the centre of a circle inscribed in a triangle.

Unit 13

Circle:	A *"circle"* is locus of a moving point in a plane which is equidistant from a fixed point. The fixed point is called *"centre"* of the circle.
Radius:	The distance from the centre of the circle to any point on the circle is called *radius* of the circle.
Perimeter:	The *perimeter* of a closed geometric figure is the sum of its sides.
circumference	The perimeter or length of the boundary of the circle is called the *circumference*.
Diameter:	A chord which passes through the centre of the circle is called *diameter* of the circle.
Arc:	A part of circumference of a circle is called **an** *arc*.
Triangle:	A plane figure formed by three straight edges as its sides is called a *triangle*.
Polygon:	A plane figure with three or more straight edges as its sides is called a *polygon*.
Regular polygon:	A figure bounded by equal straight lines which has all its angles equal is called a *regular polygon*.
Vertices:	The corners of a polygon are called its *vertices*.
Locus:	The path of an object moving according to some rule, is the *locus* of the object.
Circumscribed circle:	If a circle passes through all the vertices of a polygon the circle is said to be *circumscribed* about the polygon and the polygon is said to be *inscribed* in the circle.
Escribed circle:	If a circle touches one side of a triangle externally and the other two produced sides internally, is called *escribed* circle.
Circum circle:	The circle passing through the vertices of triangle ABC is known as *circum circle*, its radius as *circum radius* and centre as *circum centre*.
In circle:	A circle which touches the three sides of a triangle internally is known as *in-circle* its radius as *in-radius* and centre as *in-centre*.

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