Unit - 3 Number Systems

After completion of this unit, the students will be able to:

- Recognize base of a number system.
- Define number system with base 2, 5, 8 and 10
- Explain
 - Binary number system (system with base 2),
 - Number system with base 5,
 - Octal number system (system with base 8),
 - Decimal number system (system with base 10).
- Convert a number from decimal system to a system with base 2,
 5 and 8, and vice versa.
- Add, subtract and multiply numbers with base 2, 5 and 8.
- Add, subtract and multiply numbers with different bases.

3.1 NUMBER SYSTEMS

Any number can be formed with the help of 10 digits *i.e.*, 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

These numbers are called numerals and these numerals are known as 'Arabic numerals'.

3.1.1 Base of a Number System:

The number of digits involved in a number system is called the base of that number system. If a number system involves only two digits 0, 1, then base is 2. A number system, in which 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used, is a system with base 10.

Similarly, a number system in which five digits 0, 1, 2, 3 and 4 are used is a system with base 5.

3.1.2 To Define Number System with Base 2, 5, 8 and 10:

(a) Number System with Base 2:

A number system formed by two digits 0, 1 is called **Binary system** and its base is 2. This system is not used in everyday life apparently. But it is very important number system because it is used in all types of computers. Because computer stores information in the form of binary numbers so the binary system is of primary importance in the modern age of computer.

(b) Number System with Base 5:

This number system involves digits 0, 1, 2, 3 and 4. The largest digit in base 5 system is 4.

(c) Number System with Base 8:

The number system with base 8 is called octal system. In this system eight digits 0, 1, 2, 3, 4, 5, 6 and 7 are used. The largest digit in base 8 system is 7.

(d) Decimal Number System:

Decimal number system is the most popular number system in the world. In this system, ten digits (0 to 9) are used. Every number can be expressed as the sum of multiples of powers of 10 and 10 is called its base.

3.2 CONVERSIONS:

The above discussed number systems are all place value number systems. The numbers used in these systems can be converted from one system to another system. The method of successive division is used to convert a number from one system to another system. The division is performed by the base of the system in which it is being converted.

3.2.1(a) Conversion from Decimal Number System to Other Number Systems:

(i) Conversion from Decimal to Binary System:

Example 1: Convert 15 into an equivalent number with base 2 **Solution:**

$$\begin{array}{c|cccc}
2 & 15 \\
2 & 7 - 1 \\
2 & 3 - 1 \\
\hline
 & 1 - 1 \\
\end{array}$$

$$15 = (1111)_2$$

The number $(1111)_2$ will be read as one, one, one, one base 2

Example 2: Convert 541 into binary system.

Solution:

2	541
2	270 - 1
2	135 - 0
2	67 - 1
2	33 - 1
2	16 - 1
2	8 - 0
2	4 - 0
2	2 - 0
	1 - 0
	\longrightarrow

Thus, $541 = (1000011101)_2$

(ii) Conversion from Decimal System to a Number with Base 5:

Any number of decimal system can be converted into an equivalent number with base 5 as follows.

Example 3: Convert 17 into an equivalent number with base 5 **Solution:**

$$\begin{array}{c|c}
5 & 17 \\
\hline
& 3-2 \\
\longrightarrow
\end{array}$$

Thus,
$$17 = (32)_5$$

Example 4: Convert 89651 into an equivalent number with base 5 **Solution:**

5	89651
5	17930 - 1 4
5	3586 - 0
5	717 – 1
5	143 - 2
5	28 - 3
5	5 – 3
	1 - 0
	$\xrightarrow{1-0}$

Thus, $89751 = (10332101)_5$

(iii) Conversion from Decimal to Octal System (Base 8)

Example 5: Convert 824 into an equivalent number with base 8 **Solution:**

Hence, $824 = (1470)_8$

Example 6: Convert 4837 into an equivalent number with base 8 **Solution:**

Hence, $4837 = (11345)_{8}$

3.2.1(b) Conversion from other Number Systems to Decimal Number System:

(i) Conversion from Binary System to Decimal System:

For converting a number written in binary system into a number in decimal system, consider the following example.

Example 7: Convert $(1101)_2$ into equivalent number in decimal system.

Solution:
$$(1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

= $8 + 4 + 0 + 1 = 13$

(ii) Converting a Number written in Base 5 System into Decimal System:

Any number in base 5 system can be converted into base 10 system. For converting a number in base 5 into an equivalent number with base 10, consider the following example.

Example 8: Convert (413242)₅ into equivalent decimal system.

Solution:
$$(413242)_5 = 4 \times 5^5 + 1 \times 5^4 + 3 \times 5^3 + 2 \times 5^2 + 4 \times 5^1 + 2 \times 5^0$$

= $4 \times 3125 + 1 \times 625 + 3 \times 125 + 2 \times 25 + 4 \times 5 + 2 \times 1$
= $12500 + 625 + 375 + 50 + 20 + 2$
= 13572

(iii) Conversion from Octal System to Decimal System: Consider the following examples.

Example 9: Write the following octal numbers as decimal numbers.

(i)
$$(126)_{8}$$

(ii)
$$(424002)_8$$

Solution: (i) $(126)_8$

$$(126)_8 = 1 \times 8^2 + 2 \times 8^1 + 6 \times 8^0$$
$$= 1 \times 64 + 2 \times 8 + 6 \times 1$$
$$= 64 + 16 + 6 = 86$$

(ii) $(424002)_8$

$$(424002)_8 = 4 \times 8^5 + 2 \times 8^4 + 4 \times 8^3 + 0 \times 8^2 + 0 \times 8^1 + 2 \times 8^0$$

$$= 4 \times 32768 + 2 \times 4096 + 4 \times 512 + 0 + 0 + 2 \times 1$$

$$= 131072 + 8192 + 2048 + 0 + 0 + 2$$

$$= 141314$$

EXERCISE 3.1

- 1. Convert the following into decimal system.
 - (i) $(101)_2$
- (ii) $(2044)_5$
- **(iii)** (1101110)₂

- (iv) $(7016)_8$
- (v) $(2360)_8$
- **(vi)** $(1011010100)_2$

- (vii) $(1001001)_2$
- **(viii)** (3100)₅

2. Convert the following into the base system as indicated against each question.

- (i) 3025 to binary, octal and base 5 (ii) $(671)_8$ to binary and base 5
- (iii) (2006)₈ to binary and base 5 (iv) 867 to binary, octal and base 5
- (v) $(10011001)_2$ to octal and base 5

3.2.2 Adding, Subtracting and Multiplying Numbers with Base 2:

(a) Binary Number System (Base 2):

Addition: We know that in the binary number system only two digits 0 and 1 are used.

While adding, if the sum is greater than 1 then, divide the sum by 2, write the remainder and carry quotient to the next digit.

The following addition table is helpful in finding the sums in the number system with base 2.

Addition Table for Binary System

+	0	1
0	0	1
1	1	(10)

Example 10: Find the sum of $(111)_2$ and $(10)_2$.

Solution: We have $(111)_2 + (10)_2 = (1001)_2$ in the horizontal form.

In the vertical form, we write

In the second column 1 + 1 = 2 and so we carry 1 to the third column and in binary system 2 is written as $(10)_2$.

Example 11: Solve: (10110111), + (100011),

Solution: $(10110111)_2 + (100011)_2 = (11011010)_2$ in the horizontal form.

In the vertical form, we write

$$\frac{(10110111)_2}{+(100011)_2}$$

$$\frac{(11011010)_2}{(11011010)_2}$$

Subtraction:

Example 12: Find: $(101)_2 - (11)_2$

Solution:

In decimal system we borrow one "10" from the next column for subtracting a greater number from smaller number. Similarly, in binary system we borrow one "2" from the next column. In the 2nd column 1 cannot be subtracted from 0, so we borrow 1 from third column.

Example 13: Subtract $(1101)_2$ from $(10011)_2$

Solution:

Multiplication:

The numbers having base 2, we use the following multiplication table.

Multiplication Table (Base 2)

×	0	1
0	0	0
1	0	1

Example 14: Multiply $(11)_2$ by $(10)_2$

Solution:

Example 15: Solve: $(11011011)_2 \times (10101)_2$

Solution:

(b) Base 5 Number System:

Addition:

While adding, if the sum of two or more digits is greater than 5, divide the sum by 5, write the remainder and carry the quotient to the next digit.

The following addition table will be helpful in finding the sums in the number system with base 5.

Addition Table for base 5

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

The process of addition is explained by the following examples.

Example 16: Solve: $(4)_5 + (3)_5$

Solution:

4+3=7 and in the system with base 5, 7 is represented by $(12)_5$ So, $(4)_5+(3)_5=(12)_5$.

Example 17: Find the sum of $(12433)_5$ and $(31243)_5$

Solution:

Subtraction:

Example 18: Find: $(3421)_5 - (2143)_5$

Solution:

To subtract 3 from 1 is not possible so borrow a "5" from the second column and add it to the 1^{st} column i.e., 5+1=6 and then subtract 3 from 6.

In the $2^{\rm nd}$ column "1" is left behind. Now 4 cannot be subtracted from 1. Again borrow a "5" from the $3^{\rm rd}$ column and add it to the second column *i.e.*, 5+1=6 and 6-4=2. After borrowing 1, 3 is left in the $3^{\rm rd}$ column and so 3-1=2.

Multiplication:

For multiplying the number having base 5, the following multiplication table is useful.

Multiplication Table (Base 5)

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

Example 19: Multiply $(23)_5$ by $(14)_5$

Solution:

$$\begin{array}{ccccc}
 & & & & & \\
 & & (2 & 3)_5 \\
 \times & (1 & 4)_5 \\
\hline
 & 2 & 0 & 2 \\
 & 2 & 3 & 0 \\
\hline
 & (4 & 3 & 2)_5
\end{array}$$

Since $4 \times 3 = 12$, divide 12 by 5, carry the quotient 2 and write down the remainder 2.

Similarly $4 \times 2 = 8$, add 2 already carried then divide 10 by 5, carry the quotient 2 and write down the remainder 0.

Example 20: Solve: $(421)_5 \times (234)_5$

Solution:

(c) Octal Number System (Base 8)

Addition:

In octal number system, we start counting from 0 and proceed to 7. We add one more unit in 7, we get eight which is written as:

$$7 + 1 = (10)_{8}$$

It is read as, one zero with base 8

Add the following octal numbers. Example 21:

(i)
$$(6)_8 + (7)_8$$

(ii)
$$(64)_8 + (44)_8$$

$$(6)_8 + (7)_8$$
 (ii) $(64)_8 + (44)_8$ (iii) $(255636)_8 + (143576)_8$

Solution:

(i) $(6)_8 + (7)_8$

Write $(6)_8 + (7)_8$ in the vertical form.

$$\frac{(6)_8}{+(7)_8}$$

$$\frac{(15)_8}{}$$

Thus, $(6)_8 + (7)_8 = (15)_8$

(ii) $(64)_8 + (44)_8$

Write $(64)_8 + (44)_8$ in the vertical form.

Thus, $(64)_8 + (44)_8 = (130)_8$

 $(255636)_8 + (143576)_8$ (iii)

Write $(255636)_8 + (143576)_8$ in the vertical form.

Thus, $(255636)_8 + (143576)_8 = (421434)_8$

Subtraction:

Example 22: Evaluate the following:

(i)
$$(14)_8 - (6)_8$$

(ii)
$$(604)_8 - (247)_8$$

(iii)
$$(455122)_8 - (216634)_8$$

Solution: (i) $(14)_8 - (6)_8$

Write $(14)_8 - (6)_8$ in the vertical form.

$$\frac{(14)_8}{-(6)_8}$$

Thus, $(14)_8 - (6)_8 = (6)_8$

(ii) $(604)_8 - (247)_8$

Write $(604)_8 - (247)_8$ in the vertical form.

$$\frac{(604)_8}{-(247)_8}$$

$$\frac{-(335)_8}{(335)_8}$$

Thus, $(604)_8 - (247)_8 = (335)_8$

(iii) $(455122)_8 - (216634)_8$

Write $(455122)_8 - (216634)_8$ in the vertical form.

$$\frac{-\frac{(455122)_8}{-\frac{(216634)_8}{(236266)_8}}$$

Thus, $(455122)_8 - (216634)_8 = (236266)_8$

Multiplication:

Example 23: Multiply

(i)
$$(36)_8 \times (43)_8$$

(ii)
$$(446)_8 \times (213)_8$$

Solution:

(i)
$$(36)_8 \times (43)_8$$

Write $(36)_8 \times (43)_8$ in the vertical form.

Thus, $(36)_8 \times (43)_8 = (2032)_8$

(ii)
$$(446)_8 \times (213)_8$$

Write $(446)_8 \times (213)_8$ in the vertical form.

Thus, $(446)_8 \times (213)_8 = (117642)_8$

3.2.3 Adding, Subtracting and Multiplying Numbers with Different Bases

As we are familiar in our daily life with decimal number system so, in order to perform arithmetic operations or numbers with different bases (2,5,8 or 10), we first convert all the numbers into the decimal system and perform the given operations. Then the answer can be converted into base 2,5 and 8 as required.

Example 24: Find: $(100111)_2 + (4123)_5 + 567$ and express the answer in all the three number systems. (i.e., in the number system with bases 2, 5 and 10)

Solution: We convert both $(100111)_2$ and $(4123)_5$ into decimal system.

$$(100111)_{2} = 1 \times 2^{5} + 0 \times 2^{4} + 0 \times 2^{3} + 1 \times 2^{2} + 1 \times 2^{1} + 1 \times 2^{0}$$

$$= 32 + 0 + 0 + 4 + 2 + 1 = 39$$

$$(4123)_{5} = 4 \times 5^{3} + 1 \times 5^{2} + 2 \times 5^{1} + 3 \times 5^{0}$$

$$= 500 + 25 + 10 + 3 = 538$$

$$(10111)_{2} + (4123)_{5} + 567 = 39 + 538 + 567 = 1144$$

Now, we convert 1144 into the systems with base 2 and base 5.

2	1144			
2	572 - 0			
2	286 - 0		5	1144
2	143 - 0		5	228 - 4
2	71 - 1		5	45 – 3
2	35 – 1		5	9 – 0
2	17 - 1			1 – 4
2	8 – 1			
2	4 - 0		114	$4 = (14034)_5$
2	2 - 0			
	1 – 0			
0001	$(111000)_2$			
+ (4	$(123)_5 + 567 =$	$=(10001111000)_{2}$		
	-	_		
	2 2 2 2 2 2 2 2 2 2 2 2 2 4 4 4 4 4 4 4	$ \begin{array}{c cccc} 2 & 572 - 0 \\ \hline 2 & 286 - 0 \\ 2 & 143 - 0 \\ \hline 2 & 71 - 1 \\ 2 & 35 - 1 \\ \hline 2 & 17 - 1 \\ \hline 2 & 8 - 1 \\ \hline 2 & 4 - 0 \\ \hline 2 & 2 - 0 \\ \hline 1 - 0 \\ 0001111000)_2 \\ + (4123)_5 + 567 = 0 $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

Example 25: Evaluate: $(777)_8 - (2343)_5 - (1000111)_2$

And express the answer in the number system with base 2

Solution: Convert all the numbers into decimal number system.

$$(777)_{8} = 7 \times 8^{2} + 7 \times 8^{1} + 7 \times 8^{0}$$

$$= 7 \times 64 + 56 + 7 \times 1$$

$$= 448 + 56 + 7$$

$$= 511$$

$$(2343)_{5} = 2 \times 5^{3} + 3 \times 5^{2} + 4 \times 5^{1} + 3 \times 5^{0}$$

$$= 250 + 75 + 20 + 3$$

$$= 348$$

$$(1000111)_{2} = 1 \times 2^{6} + 0 \times 2^{5} + 0 \times 2^{4} + 0 \times 2^{3} + 1 \times 2^{2} + 1 \times 2^{1} + 1 \times 2^{0}$$

$$= 64 + 4 + 2 + 1 = 71$$

$$(777)_{8} - (2343)_{5} - (1000111)_{2} = 511 - 348 - 71$$

$$= 511 - 419$$

$$= 92$$

Now convert 92 into binary system

$$\begin{array}{c|cccc}
 & 2 & 92 \\
\hline
 & 2 & 46 & -0 \\
\hline
 & 2 & 23 & -0 \\
\hline
 & 2 & 11 & -1 \\
\hline
 & 2 & 5 & -1 \\
\hline
 & 2 & 2 & -1 \\
\hline
 & 1 & -0 \\
\hline
\end{array}$$

$$\therefore$$
 92 = (1011100)₂

EXERCISE 3.2

- 1. Solve:
- (i) $(101)_2 + (111)_2$ (ii) $(11001000111)_2 + (1010110111)_2$
- (iii) $(11011)_2 (10000)_2$ (iv) $(111011)_2 \{(1010)_2 + (1001)_2\}$
- (v) $(11111111)_2 \times (11011)_2$ (vi) $(2244)_5 + (4433)_5$
- (vii) $(340102)_5 + (230124)_5$ (viii) $(100001)_5 (33322)_5$
- (ix) $(44143)_5 \times (23023)_5$ (x) $(43230)_5 \times (2412)_5$
- (xi) $(5631)_8 + (2456)_8$ (xii) $(7541)_8 (5675)_8$
- (xiii) $(4672)_8 \times (507)_8$ (xiv) $(2465)_8 \times (465)_8$
- (xv) $635 \{(2244)_5 (1243)_5 (110111)_2\}$

2. Evaluate and express the answer with bases 2, 5 and 8.

(i)
$$(75)_8 + (1342)_5 + (100111)_2$$

(ii)
$$248 + (3124)_5 - (110110)_7$$

(iii)
$$(563)_8 - \{(4433)_5 - (2134)_5 - (111011)_2\}$$

(iv)
$$(3344)_5 + \{(4101)_5 + (217)_8 + (1010101)_2 - (11011)_2\}$$

(v)
$$(6767)_8 - \{(101111101)_2 - (4213)_5 + (1423)_5 - (1110111001)_2\}$$

(vi)
$$(1423)_5 \times (110011)_2 - (243)_5$$

(vii)
$$(1010111010)_2 \times (40401)_5 + (4301)_5 \times (111001)_2$$

(viii)
$$\{(3404)_5 + (1100101)_2\} \{(3404)_5 - (1100101)_2\}$$

(ix)
$$\{(467)_8 + (101110011)_2\} \times \{(467)_8 + (3004)_5\}$$

(x)
$$\{(31234)_5 + (10110111)_2\} \{2459 - (1342)_5\}$$

REVIEW EXERCISE 3

1. Four options are given against each statement. Encircle the correct one.

i.	Only $0, 1, 2, 3$ and 4 are used in:		
	(a) binary system	(b)	octal system
	(c) decimal system	(d)	system with base 5
::	The place value of 10^3 is.		

- ii. The place value of 10° is:
 - (a) 30 (b) 300 (c) 100 (d) 1000
- iii. $(100)_2$ in decimal system is written as:
- (a) 2 (b) 4 (c) 7 (d) 10
- iv. $(304)_5$ in decimal system is written as:
- (a) 75 (b) 79 (c) 30 (d) 34 v. $(11)_2 + (10)_2 = ?$
- (a) $(110)_2$ (b) $(101)_2$ (c) $(111)_2$ (d) $(11)_2$
- vi. $(3)_5 \times (4)_5 = ?$ (a) $(22)_5$ (b) $(33)_5$ (c) $(34)_5$ (d) $(12)_5$
- vii. $(400)_5 (33)_5 = ?$ (a) $(312)_5$ (b) $(367)_5$ (c) $(312)_{10}$ (d) $(367)_{10}$
- **2.** Answer the following questions.
- i. Define the binary system
- ii. Write the digits used in octal system.
- iii. Define decimal number system.
- iv. Which is the biggest digit used in system with base 2?

- 3. Express the following as decimal numbers.
 - i. $(101)_2$ ii. $(1000)_2$ iii. $(2003)_5$ iv. $(3276)_8$ v. $(1134)_5$
- 4. Convert the following into number with base 5 and octal system.
 - i. 154 ii. 820
- iii. 2640
- iv. 51605
- **v.** 898

- **5.** Solve the following:
 - i. $(11001)_2 + (101)_2$
- ii. $(100111)_2 + (10111)_2$
- iii. $(10000)_2 (111)_2$
- **6.** Evaluate the following:
 - i. $(21304)_5 + (2003)_5$
- ii. $(4001)_5 (302)_5$
- iii. $(2442)_5 + (4043)_5$
- iv. $(212)_5 \times (34)_5$
- **7.** Solve the following:
 - i. $(546)_8 + (327)_8$
- ii. $(7000)_8 (4456)_8$
- iii. $(7643)_{s} \times (2346)_{s}$
- iv. $(467)_8 \times (433)_8$
- 8. Evaluate and express the answer into decimal number system.
 - i. $(2273)_8 \{(104)_5 + (42)_5\}$
- ii. $\{(80)_{10} + (241)_5\} + \{(34)_5 (111)_2\}$
- iii. $[278819 \{60065 ((202)_5 + (101)_2)\}]$

SUMMARY

- The number system with base 2 is also called "Binary number system".
- All the numbers in binary number system are represented by only two digits 0 and 1.
- All the binary numbers can be represented by the sum of multiples of power of base 2.
- In base 5 number system, five digits 0, 1, 2, 3 and 4 are used to represent numbers in the system.
- All the base 5 numbers can be represented by the sum of multiples of power of base 5
- The number system with base 8 is also called "Octal number system".
- In octal number system eight digits 0, 1, 2, 3, 4, 5, 6 and 7 are used to represent numbers in the system.
- All the octal numbers can be represented by the sum of multiples of power of base 8.
- In decimal number system numbers are represented by ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
- Decimal number system is a place value system in which value of each position is some power of 10 starting from zero onwards.
- To convert a number from one system to another system, the method of successive division by the base is used.