Unit - 8 ▶ Practical Geometry

After completion of this unit, the students will be able to:

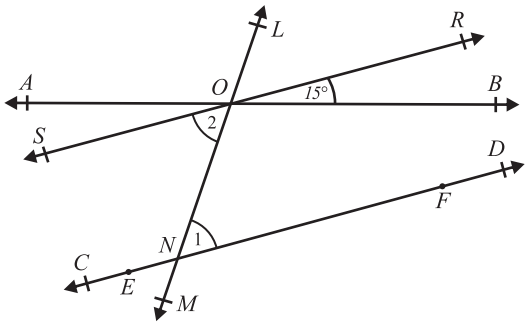
- Define and depict two converging (non-parallel) lines and find the angle between them without producing the lines.
- Bisect the angle between the two converging lines without producing them.
- Construct a square
  - When its diagonal is given.
  - When the difference between its diagonal and side is given.
  - When the sum of its diagonal and side is given.
- Construct a rectangle
  - When two sides are given.
  - When the diagonal and a side are given.
- Construct a rhombus
  - When one side and the base angle are given.
  - When one side and a diagonal are given.
- Construct a parallelogram
  - When two diagonals and the angle between them is given.
  - When two adjacent sides and the angle included between them is given.
- Construct a kite
  - When two unequal sides and a diagonal are given.
- Construct a regular pentagon
  - When a side is given.
- Construct a regular hexagon
  - When a side is given.
- Construct a right angled triangle
  - When hypotenuse and one side are given.
  - When hypotenuse and the vertical height from its vertex to the hypotenuse are given.

8.1 Define and depict two Converging (non-parallel) lines and find the angle between them without producing the lines

8.1.1 Definition:

Lines intersecting at a single point are called converging lines.

In the following figure, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are converging lines and $\overleftrightarrow{LM}$ is a transversal intersecting these lines. Find the angle between converging lines.



Steps of construction:

i. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two converging lines and $\overleftrightarrow{LM}$ is the transversal which intersects these lines at point O and N.

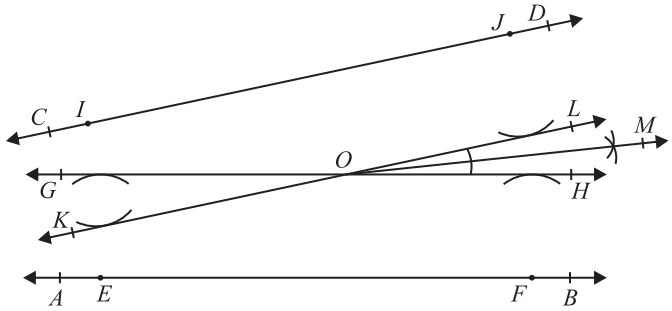
ii. Draw $m\angle 2 = m\angle 1$ with compass and straightedge. Thus $\overleftrightarrow{SOR}$ is parallel to $\overleftrightarrow{CD}$.

iii. Since $\overleftrightarrow{CD}$ and $\overleftrightarrow{SR}$ are parallel, therefore $m\angle BOR$ is the required angle.

iv. Hence, angle between converging lines is $15°$ which is measured by using protractor.

8.1.2 Bisect the angle between two converging lines without producing them

We can find the angle bisector of two converging lines by performing the following steps:



Steps of construction:

i. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two converging lines.

ii. Draw two arcs of same radius from points E and F above $\overleftrightarrow{AB}$ by using compass and draw $\overleftrightarrow{GH}$ touching these arcs.

iii. Also draw two arcs of same radius from points I and J below $\overleftrightarrow{CD}$ by using compass and draw $\overleftrightarrow{KL}$ touching these arcs.

iv. $\angle HOL$ is the angle between the two convergent lines.

v. Draw the bisector $\overrightarrow{OM}$ of $\angle HOL$ which is the required bisector of given converging lines.

8.1.3 Construct a square

(a) When its diagonal is given.

Example 1:

Draw a square $ABCD$ such that its diagonal is $4cm$

Solution:

One of the diagonals of the square $ABCD$ is $\overline{BD}$ and $m\overline{BD} = 4cm$.

[Note: In a square both the diagonals are of same length]

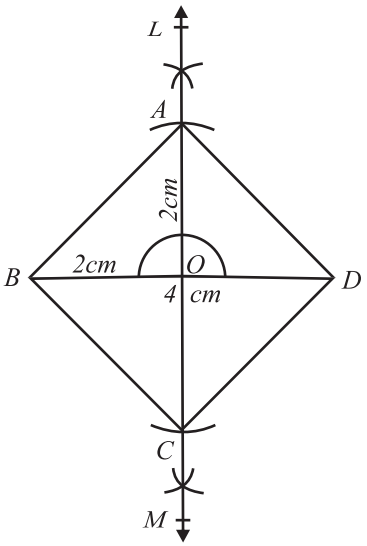
Steps of construction:

i. Draw the diagonal $m\overline{BD} = 4cm$.

ii. Draw a perpendicular bisector $\overleftrightarrow{LM}$ of the diagonal $\overline{BD}$ cutting it at point O.

iii. With O as centre and radius $m\overline{OB}$, draw arcs cutting $\overleftrightarrow{LM}$ at A and C.

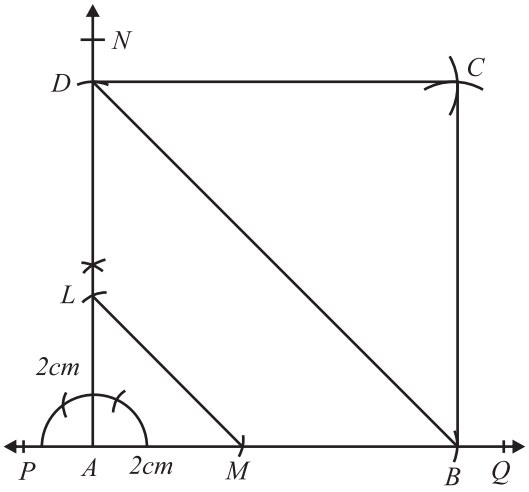
iv. Join A with B and D, and C with B and D, which gives the required square $ABCD$



(b) When the difference between its diagonal and side is given

Example 2:

Draw a square $ABCD$ when the difference between its diagonal and side is equal to $2cm$.



Solution:

Steps of construction:

i. Draw $\overleftrightarrow{PQ}$ and mark a point as A on it.

ii. Construct $m\angle QAN = 90°$ at A.

iii. Draw two arcs of radius $2cm$ and centre at A which intersects $\overrightarrow{AQ}$ at M and $\overrightarrow{AN}$ at L.

iv. Draw an arc of radius = $m\overline{LM}$ and centre at *M* which intersects $\overrightarrow{AQ}$ at B.

v. Draw an arc of radius = $m\overline{AB}$ and centre at *A* which intersects $\overrightarrow{AN}$ at D.

vi. Draw two arcs each of radius = $m\overline{AB}$, one centre at *B* and second centre at D. These arcs will intersect at point C.

vii. Join C with D and B.

Hence, $ABCD$ is the required square.

(c) When the sum of its diagonal and side is given

Example 3:

Draw a square $ABCD$ when the sum of its diagonal and side is equal to $3cm$.

Solution:

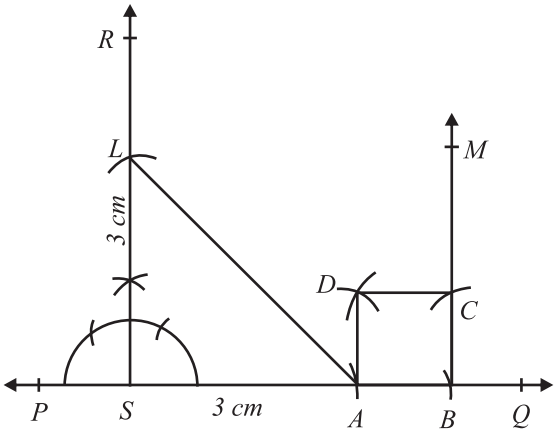
Steps of construction:

i. Draw $\overleftrightarrow{PQ}$ and mark a point as S on it.

ii. Construct $m\angle QSR = 90°$at point S.

iii. Draw an arc of radius $3cm$ and centre at S intersecting $\overrightarrow{SR}$ at L.

iv. Draw an arc of radius $3cm$ and centre at S intersecting $\overrightarrow{SQ}$ at A.



v. Draw an arc of radius $= m\overline{AL}$ and centre at S which intersects $\overrightarrow{SQ}$ at B. $\overline{AB}$ is the side of the required square.

vi. Draw perpendicular $\overrightarrow{BM}$ at B.

vii. Draw an arc of radius $m\overline{AB}$ and centre at B which intersects $\overrightarrow{BM}$ at C.

viii. Draw two arcs, each of radius $m\overline{AB}$, one with centre at A and second with centre at C which intersects at D.

ix. Join C with D and D with A

Hence, $ABCD$ is the required square.

8.1.4 Construct a rectangle

(a) When two sides are given

Example 4:

Construct a rectangle $ABCD$ in which $m\overline{AB} = 4\ cm$ and $m\overline{BC} = 5\ cm$.

Solution:

Steps of construction:

i. Draw $m\overline{AB} = 4cm$.

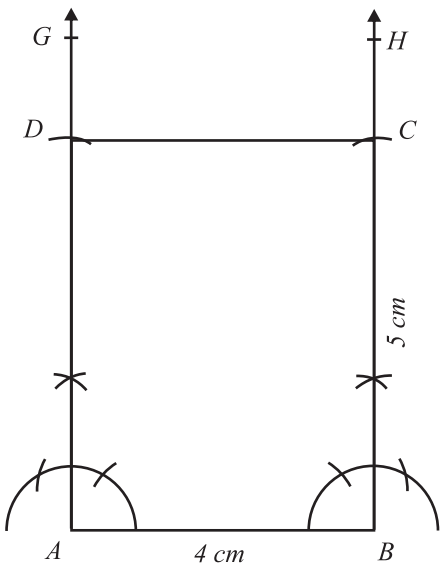
ii. Construct $m\angle A = m\angle B = 90°$ and draw $\overrightarrow{AG}$ and $\overrightarrow{BH}$.

iii. Draw an arc with centre at A and of radius $5cm$ which intersects the $\overrightarrow{AG}$ at point D.

iv. Draw an arc with centre at B and of radius $5cm$ which intersects the $\overrightarrow{BH}$ at point C.

v. Join C with D.

Hence, $ABCD$ is the required rectangle.

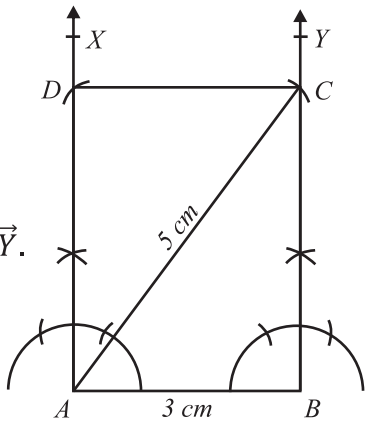


Note: Sum of interior angles of a quadrilateral is equal to 360°

(b) When the diagonal and a side are given

Example 5:

Construct a rectangle $ABCD$ when $m\overline{AB} = 3cm$ and $m\overline{AC} = 5cm$



Solution:

Steps of construction:

i. Draw $m\overline{AB} = 3cm$.

ii. Construct $m\angle A = m\angle B = 90°$ and draw $\overrightarrow{AX}$ and $\overrightarrow{BY}$.

iii. With centre at *A* and radius 5 *cm* draw an arc which intersects $\overrightarrow{BY}$ at the point C.

iv. With centre at B and radius $5cm$ draw an arc which intersects $\overrightarrow{AX}$ at the point D and joint C and D.

Hence, $ABCD$ is the required rectangle.

8.1.5 Construct a rhombus

(a) When one side and the base angle are given.

Example 6: Construct a rhombus $PQRS$ when the $m\overline{PQ} = 4cm$ and $m\angle P = 45°$

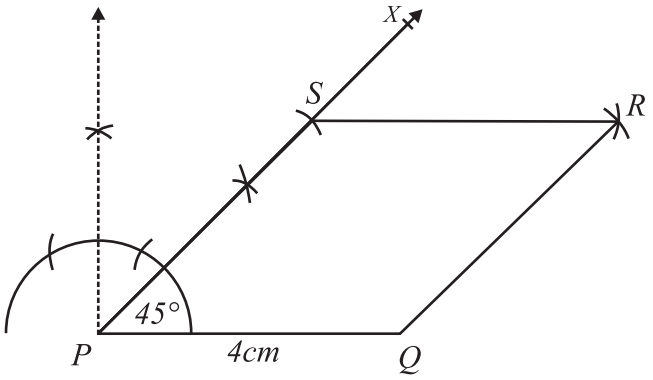
Solution:

Steps of construction:

i. Draw $m\overline{PQ} = 4cm$.

ii. Construct $m\angle P = 45°$ and draw $\overrightarrow{PX}$.

iii. Draw an arc with centre at P and radius $4cm$ which intersects $\overrightarrow{PX}$ at S.



iv. Draw an arc with centre at S and radius $4cm$.

v. Draw an arc with centre at Q and radius $4cm$ which intersects the previous arc drawn from S at R.

vi. Join R with S and Q.

Hence, $PQRS$ is the required rhombus.

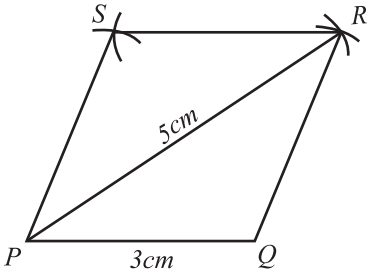
(b) When one side and a diagonal are given.

Example 7:

Construct a rhombus $PQRS$, when $m\overline{PQ} = 3cm$ *and* $m\overline{PR} = 5cm$.

Solution:

Steps of construction:



i. Draw $m\overline{PQ} = 3cm$.

ii. Draw an arc with centre at P and radius $5cm$.

iii. Draw an arc with centre at Q and radius $3cm$ which intersects the previous arc at R.

iv. Draw an arc with centre at R and radius $3cm$.

v. Draw an arc with centre at P and radius $3cm$ which intersects the previous arc at S.

vi. Join Q with R, R with S and P with S.

Hence, $PQRS$ is the required rhombus.

8.1.6 Construct a parallelogram

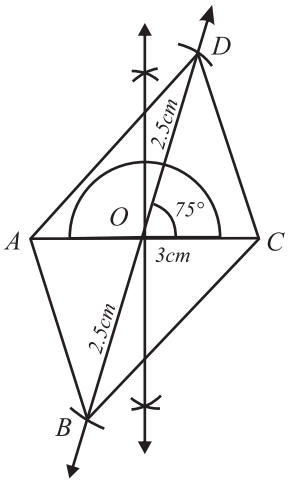
(a) When two diagonals and the angle between them is given.

Example 8:

Construct a parallelogram $ABCD$ whose diagonals are $3cm$ and $5cm$ and the angle between them is $75°$.

Solution:

Steps of construction:



i. Draw the diagonal $m\overline{AC} = 3cm$.

ii. Bisect $\overline{AC}$ with O as the midpoint.

iii. Construct an angle $75°$ at the point O and extend the line on both sides.

iv. From O, draw an arc of radius $2.5cm$ on both sides of $\overline{AC}$ to cut the above line at B and D.

v. Join A with B and D.

vi. Join C with B and D.

Hence, $ABCD$ is the required parallelogram.

(b) When two adjacent sides and the angle included between them are given

Example 9:

Construct a parallelogram $PQRS$ when $m\overline{PQ} = 4cm$, $m\overline{PS} = 7cm$ and included angle between these sides is $m\angle QPS = 60°$.

Solution:

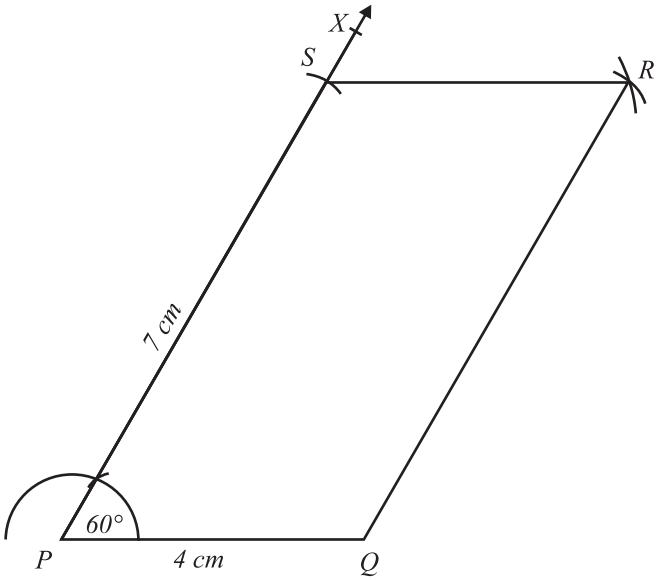
Steps of construction:

i. Draw a line segment $PQ = 4cm$.

ii. Construct $m\angle QPX = 60°$ at point P.

iii. Draw an arc with centre at P and radius $7cm$ which intersects $\overrightarrow{PX}$ at point S.

iv. Draw an arc with centre at Q and radius $7cm$ above point Q.



v. Draw an arc with centre at S and radius $4cm$ which intersects the arc drawn from point Q at R.

vi. Join R with S and Q to R to form the required parallelogram $PQRS$.

8.1.7 Construct a kite when two unequal sides and a diagonal are given

Example 10:

Construct a kite $PQRS$ when $m\overline{PQ} = 4cm$, $m\overline{QR} = 6cm$ and the length of the longer diagonal is $m\overline{PR} = 8cm$.

Solution:

Steps of construction:

i. Draw $m\overline{PQ} = 4cm$.

ii. Draw an arc with centre at Q and radius $6cm$.

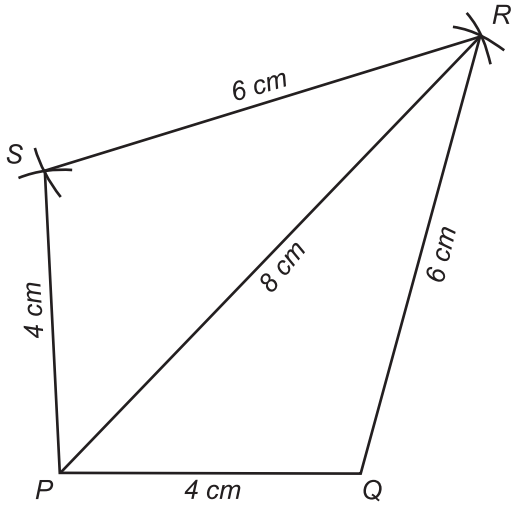
iii. Draw an arc with centre at P and radius $8cm$. It intersects the previous arc at point R.

iv. Draw an arc with centre P and radius $4cm$ above P.

v. Draw an arc with centre at R and radius $6cm$ which intersects the arc drawn from P at S.

vi. Join R with Q and S and P with S.

Hence, $PQRS$ is the required kite



8.1.8 Construct a regular pentagon when a side is given

Example 11: Construct a regular pentagon $PQRST$ when $m\overline{PQ} = 4cm$

Solution:

Steps of construction:

i. Draw $m\overline{PQ} = 4cm$.

ii. Construct $m\angle P = m\angle Q = 108°$.

[NOTE: Each interior angle of a regular pentagon is equal to $108°$.]

iii. Draw an arc with centre at P and radius $4cm$ which intersects $\overrightarrow{PX}$ at T.

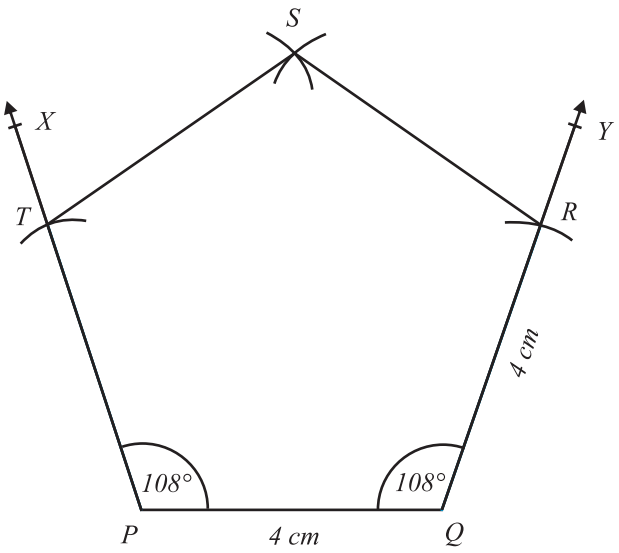
iv. Draw an arc with centre at Q and radius $4cm$ which intersects $\overrightarrow{QY}$ at R.

v. Draw an arc with centre at R and radius $4cm$.

vi. Draw an arc with centre at T and radius $4cm$.

It intersects the arc drawn from point R at the point S.

vii. Join R with S and T with S.



Hence, $PQRST$ is the required regular pentagon.

8.1.9 Construct a regular hexagon when a side is given

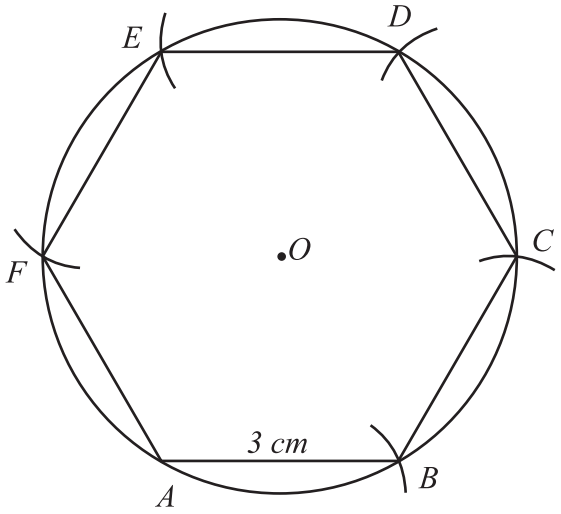
Example 12:

Construct a regular hexagon $ABCDEF$ when $m\overline{AB} = 3cm$

Solution:

Steps of construction:

i. Draw a circle of radius $3cm$ with centre at O.



ii. Take a point A on the circle, draw an arc on the circle with centre A and radius $3cm$. Label it as B.

iii. Take B as the centre and radius $3cm$ draw an arc on the circle, mark it as C.

iv. Take C as the centre and radius $3cm$ draw an arc on the circle, mark it as D.

v. Take D as the centre and radius $3cm$ draw an arc on the circle, mark it as E.

vi. Take E as the centre and radius $3cm$ draw an arc on the circle, mark it as F.

vii. Join B with C, C with D, D with E, E with F and F with A.

Hence, $ABCDEF$ is the required regular hexagon.

Note: Each interior angle of a rectangular hexagon is equal to 120°

EXERCISE 8.1

1. Construct a square $ABCD$ when a diagonal $m\overline{AC} = 4.5cm$.
2. Construct a square $PQRS$ when its diagonal is $4cm$ more than its side.
3. Construct a square $PQRS$, when the sum of the diagonal and a side of the square is $8cm$.
4. Construct a rectangle $ABCD$ when $m\overline{AB} = 4cm$ and $m\overline{BC} = 6cm$.
5. Construct a rectangle $ABCD$, when the $m\overline{AB} = 5.5cm$ and $m\overline{AC} = 8cm$
6. Construct a rhombus $KLMN$, when the $m\overline{KL} = 5cm,\ m\angle K = 75°$
7. Construct a rhombus $STUV$, when $m\overline{ST} = 6cm$ and $m\overline{SU} = 9cm$
8. Construct a parallelogram $ABCD$ with diagonals $6cm$ and $8cm$ and the angle between them $70°$.
9. Construct a parallelogram $DEFG$ where $m\overline{DE} = 5.5cm$, $m\overline{EF} = 6.5cm$ and $m\angle E = 60°$.
10. Construct a kite $DEFG$ where $m\overline{DE} = 4cm$, $m\overline{EF} = 8cm$ and the length of the longer diagonal is $m\overline{DF} = 10cm$.
11. Construct a regular pentagon $ABCDE$, where $m\overline{AB} = 3.2cm$.
12. Construct a regular hexagon $STUVWX$, where $m\overline{ST} = 3cm$.

8.2 Construction of a Right angled triangle

(a) Construct a right angled triangle when hypotenuse and one side are given

Example 1: Construct a right angled triangle ABC, when $m\overline{AB} = 5cm$, $m\overline{AC} = 7cm$ and $m\angle B = 90°$

Solution

Steps of construction:

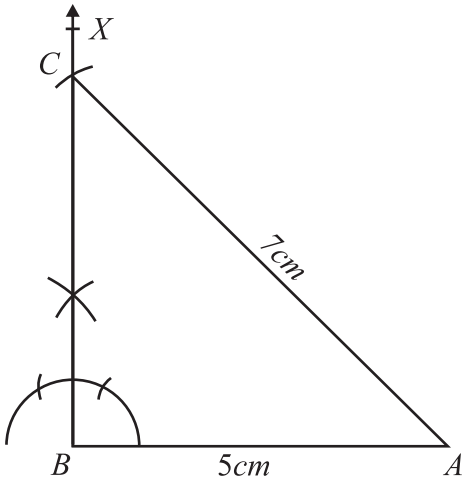
i. Draw $m\overline{AB} = 5cm$.

ii. Construct $m\angle B = 90°$. Draw $\overrightarrow{BX}$

iii. Take A as the centre and radius $7cm$. Draw an arc on intersecting $\overrightarrow{BX}$ at C.

iv. Join A with C.

Hence, ABC is the required right angled triangle.



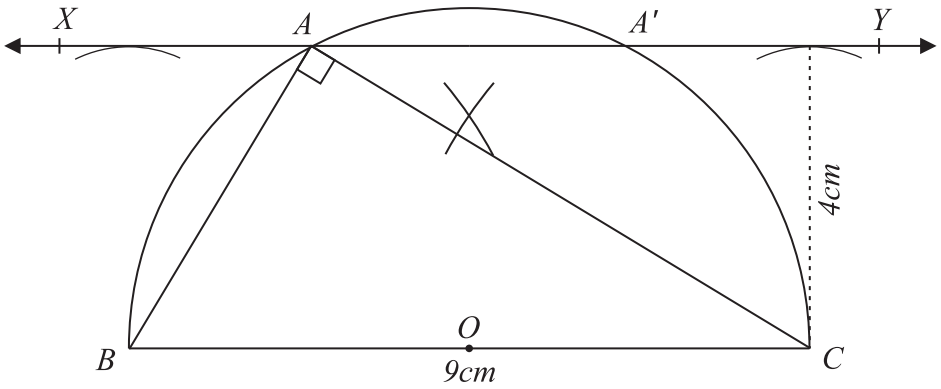
(b) Construct a right angled triangle when hypotenuse and the vertical height from its vertex to the hypotenuse are given

Example 2:

Construct a right angled triangle ABC, when hypotenuse $m\overline{BC} = 9cm$ and perpendicular from vertex A to $\overline{BC}$ is $4cm$.

Solution:

Steps of construction:



i. Draw $m\overline{BC} = 9cm$.

ii. Bisect the $\overline{BC}$ at point O with the help of compass.

iii. Draw a semi circle taking point O as centre.

iv. Draw two arcs of radius $4cm$ taking points B and C as centre above $\overline{BC}$.

v. Draw $\overleftrightarrow{XY}$ touching the two arcs which intersects the semi circle at points A and A'.

vi. Join A with B and C.

ΔABC is the required right angled triangle at angle A.

EXERCISE 8.2

1. Construct following right angled triangles when:

a. Hypotenuse = $8.5cm$ and length of a side is $6cm$.

b. Hypotenuse = $6cm$ and length of a side is $3cm$.

c. Hypotenuse = $5cm$ and length of a side is $2.5cm$.

2. Construct a right angled triangle ABC, when $m\overline{AB} = 4.5cm$, $m\overline{BC} = 5.5cm$ and $m\angle B = 90°$.

3. Construct a right angled triangle PQR, when $m\overline{QR} = 8cm$, $m\overline{PQ} = 5cm$ and $m\angle Q = 90°$.

4. Construct a right angled triangle LMN, when hypotenuse $m\overline{MN}$ = 8cm and perpendicular from vertex L to $\overline{MN}$ is 3.5cm.

REVIEW EXERCISE 8

1. Four options are given against each statement. Encircle the correct one.

i. A polygon with sum of measure of interior angles equal to $360°$ is called:

(a) triangle **(b)** quadrilateral **(c)** pentagon **(d)** hexagon

ii. In a square, the diagonals:

(a) bisect each other **(b)** do not intersect

(c) are of unequal lengths **(d)** do not bisect each other

iii. In regular pentagon, the measure of an interior angle is:

(a) $100°$ **(b)** $108°$ **(c)** $116°$ **(d)** $124°$

iv. In a rectangle, the diagonals:

(a) bisect each other **(b)** are perpendicular to each

(c) are parallel to each other **(d)** none of the above

v. In a rhombus, the diagonals:

(a) bisect the vertex angle **(b)** are of equal length

(b) are not perpendicular to each other **(d)** all of the above

vi. Square is a:

(a) pentagon **(b)** quadrilateral

(c) triangle **(d)** none of the above

vii. The measure of one interior angle of a regular hexagon is:

(a) $108°$ **(b)** $120°$ **(c)** $140°$ **(d)** $170°$

viii. If the measure of three angles of a quadrilateral are $108°, 128°$ and $76°$, then measure of its fourth angle is:

(a) $48°$ **(b)** $88°$ **(c)** $98°$ **(d)** $108°$

2. Construct the following:

i. Square $PQRS$ such that $m\overline{RS} = 4cm.$

ii. Square $ABCD$ such that $m\overline{AC} = 3.5cm.$

iii. Square $WXYZ$, when the difference of its diagonal and side is $5cm.$

iv. Square $PQRS$, when the sum of its diagonal and side is $8cm.$

v. Rectangle $ABCD$ in which $m\overline{AB} = 5.5cm$ and $m\overline{BC} = 8cm.$

vi. Rectangle $LMNO$, when $m\overline{LM} = 4cm$ and $m\overline{LN} = 6cm$

vii. Rhombus $PQRS$, when $m\overline{PQ} = 5.5cm$ and $m\angle P = 75°$.

viii. Parallelogram $ABCD$ whose diagonals are $5cm$ and $9cm$ and the included angle is $80°$.

ix. Parallelogram $UVWX$ with sides $m\overline{UV} = 8cm, m\overline{UX} = 5cm$ and $m\angle U = 60°$.

x. Kite $ABCD$ with $m\overline{AB} = 4cm, m\overline{BC} = 6cm$ and the length of the longer diagonal is $m\overline{AC} = 7cm.$

xi. Regular pentagon $GHIJK$, when $m\overline{GH} = 4cm.$

SUMMARY

- Quadrilateral is a 4-sided polygon which has the sum of interior angles equal to $360°$.
- Coverging lines are non-parallel lines and these lines meet at a single point.
- Diagonals of a rectangle, a square, a parallelogram and a rhombus bisect each other.
- Diagonals of a square and a rhombus bisect each other at $90°$.
- Diagonals of a square and a rectangle are of equal lengths.
- In a regular hexagon, the sum of measures of interior angles is $720°$ and the measure of each interior angle is $120°$.
- In a regular pentagon, the sum of measures of interior angles is $540°$ and the measure of each interior angle is $108°$.